Autumn Block 3

Fractions A

Small steps

Step 1	Equivalent fractions and simplifying
Step 2	Equivalent fractions on a number line
Step 3	Compare and order (denominator)
Step 4	Compare and order (numerator)
Step 5	Add and subtract simple fractions
Step 6	Add and subtract any two fractions
Step 7	Add mixed numbers
Step 8	Subtract mixed numbers

Small steps

Step 9	Multi-step problems

Equivalent fractions and simplifying

Notes and guidance

In this small step, children build on prior knowledge of equivalent fractions to recognise when fractions are, and are not, in their simplest form.

Children use their understanding of common factors to simplify fractions. They learn that when the numerator and denominator have no common factors greater than 1, the fraction is in its simplest form.

The step begins with fractions with one common factor (greater than 1) and moves on to fractions with several common factors. Children are encouraged to look for the greatest possible number to divide by, but also understand that simplification can be performed in more than one step.

Pictorial representations and fraction walls can be used to support understanding.

Things to look out for

- Children may partially simplify a fraction instead of finding the simplest form, for example $\frac{6}{24} = \frac{3}{12}$
- When simplifying mixed numbers, children may divide the whole number as well as the numerator and denominator.

Key questions

- What are the common factors of ______ and ______?
- Why is it better to identify the greatest possible number that both the numerator and denominator can be divided by?
- Does the simplified fraction have the same value?
- Do the numerator and denominator have any more common factors?
- How can you tell if a fraction is in its simplest form?
- When simplifying a mixed number, why does the integer not change?

Possible sentence stems

- Both the numerator and the denominator can be divided by ______
- To simplify the fraction, I will divide the numerator and denominator by ______
- ______ in its simplest form is ______

National Curriculum links

- Use common factors to simplify fractions; use common multiples to express fractions in the same denomination

Equivalent fractions and simplifying

Key learning

- Here are some fractions.

$\frac{4}{5}$ $\frac{30}{60}$ $\frac{7}{8}$ $\frac{42}{48}$ $\frac{2}{6}$ $\frac{1}{2}$ $\frac{8}{10}$ $\frac{16}{48}$

Find the pairs of equivalent fractions.

- Jack uses multiplication to find equivalent fractions.

$$\frac{3}{5} \overset{\times 3}{\underset{\times 3}{=}} \frac{9}{15}$$

Use Jack's method to complete the equivalent fractions.

- $\frac{4}{5} = \frac{\square}{20}$
- $\frac{4}{5} = \frac{20}{\square}$
- $\frac{\square}{7} = \frac{9}{21}$
- $\frac{4}{7} = \frac{\square}{21}$

- Use division to write the fractions in their simplest form.

- $\frac{12}{15} = \frac{4}{\square}$
- $\frac{12}{20} = \frac{\square}{5}$
- $\frac{16}{24} = \frac{2}{\square}$
- $\frac{10}{12} = \frac{\square}{\square}$
- $\frac{6}{30} = \frac{\square}{\square}$
- $\frac{24}{40} = \frac{\square}{\square}$

- Esther and Kim are simplifying fractions.

Esther

$$\frac{8}{12} \overset{\div 2}{\underset{\div 2}{=}} \frac{4}{6} \overset{\div 2}{\underset{\div 2}{=}} \frac{2}{3}$$

Kim

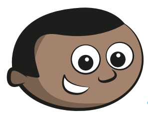
$$\frac{8}{12} \overset{\div 4}{\underset{\div 4}{=}} \frac{2}{3}$$

What is the same? What is different?

Use one of their methods to simplify the fractions.

$\frac{2}{12}$ $\frac{4}{12}$ $\frac{6}{12}$ $\frac{6}{24}$ $\frac{8}{24}$ $\frac{16}{24}$

- Mo is simplifying $2\frac{4}{10}$



$2\frac{4}{10} = 2\frac{2}{5}$

Use Mo's method to simplify the mixed numbers.

$3\frac{4}{10}$ $4\frac{12}{20}$ $6\frac{16}{30}$ $2\frac{16}{40}$

Equivalent fractions and simplifying

Reasoning and problem solving

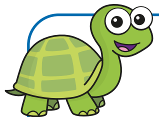
Tom and Aisha are simplifying an improper fraction.

Tom	Aisha
$\frac{36}{8} = 4\frac{4}{8} = 4\frac{1}{2}$	$\frac{36}{8} = \frac{9}{2} = 4\frac{1}{2}$

Whose method do you prefer?

Explain your answer.

various answers



Tiny is simplifying $4\frac{12}{16}$

$$4\frac{12}{16} = 1\frac{3}{4}$$

Explain Tiny's mistake.

Tiny has divided the whole number by 4 instead of just simplifying the fraction.

Here are some fractions.

$\frac{5}{15}$	$\frac{2}{4}$	$\frac{4}{16}$	$\frac{8}{16}$
$\frac{5}{10}$	$\frac{3}{9}$	$\frac{6}{12}$	$\frac{2}{8}$

Which of the fractions:

- simplify to $\frac{1}{2}$
- simplify to $\frac{1}{3}$
- simplify to $\frac{1}{4}$?

What patterns can you see?

What is the relationship between the numerator and the denominator?

Identify three more fractions that could go in each list.

simplifies to $\frac{1}{2}$:

$\frac{2}{4}, \frac{8}{16}, \frac{5}{10}, \frac{6}{12}$

simplifies to $\frac{1}{3}$:

$\frac{5}{15}, \frac{3}{9}$

simplifies to $\frac{1}{4}$:

$\frac{4}{16}, \frac{2}{8}$

multiple possible answers

Equivalent fractions on a number line

Notes and guidance

In this small step, children use number lines to count forwards and backwards in fractions and to find equivalent fractions.

Children start by revising counting fractions above 1 on a number line to ensure they are able to count in fractions accurately. Using a number line clearly shows that finding equivalent fractions does not change the value of the fraction. Encourage children to draw extra intervals on number lines to support them in placing the fractions. Number lines can also be used to support children in finding the difference between fractions. This will be revised later in the block when adding and subtracting fractions.

Encourage children to spot patterns on number lines when simplifying, rather than thinking about fractions individually.

Things to look out for

- Children may find it difficult to place a fraction on the number line when the denominator is greater than the value of the divisions on the number line.
- When crossing 1, children may not be confident in converting mixed numbers/improper fractions.

Key questions

- How many intervals are there on the number line? What is each interval worth?
- What equivalent fractions have you found?
- Is this fraction in its simplest form? How do you know?
- Can you divide the number line into more intervals to place the fractions more accurately?
- How will you place one sixteenth on a number line that is counting in eighths?
- Which fraction was the easiest/hardest to label? Why?

Possible sentence stems

- From my number line, I can see that ______ is equivalent to ______
- When I count in eighths, I can change ______ into ______ because they are equivalent.

National Curriculum links

- Use common factors to simplify fractions; use common multiples to express fractions in the same denomination

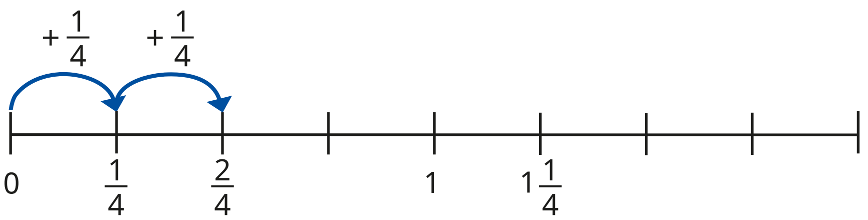
Equivalent fractions on a number line

Key learning

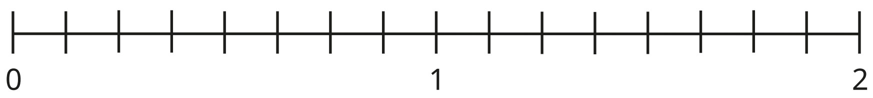
- Jack is counting in quarters.

  He writes each number on a number line.

  Complete the number line.

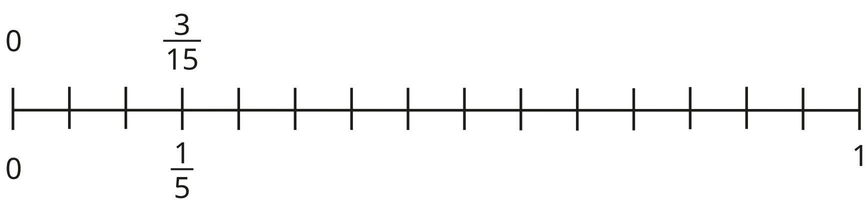


- Use the number line to count forward in eighths.



  Which of the fractions can be simplified?

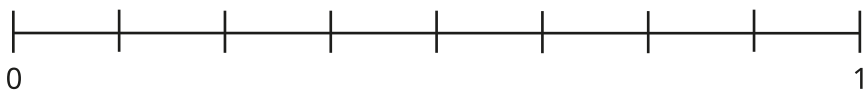
- Count in fifteenths on this number line and then write the fractions in their simplest form.



  What patterns can you see?

- Label the fractions on the number line.

  $\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{8}$ $\frac{5}{8}$ $\frac{7}{8}$

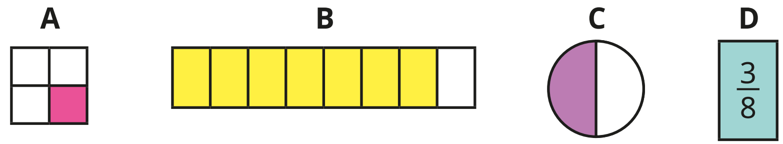


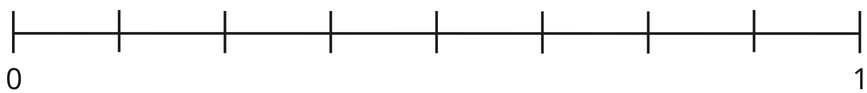
  What is the difference between the greatest and smallest fraction?

- Label the fractions on the number line.

  $\frac{3}{5}$ $\frac{7}{10}$ $\frac{24}{30}$ $\frac{1}{5}$ $\frac{3}{10}$

- Label A, B, C and D on the number line.





Equivalent fractions on a number line

Reasoning and problem solving

Rosie is counting back in tenths.

She starts at $2\frac{1}{10}$ and counts back 7 tenths.

What number does Rosie end on?

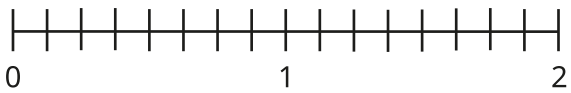
Show this on a number line.

Simplify the fraction.

$1\frac{4}{10} = \frac{14}{10}$

$1\frac{2}{5} = \frac{7}{5}$

How many ways can you show a difference of one quarter on the number line?



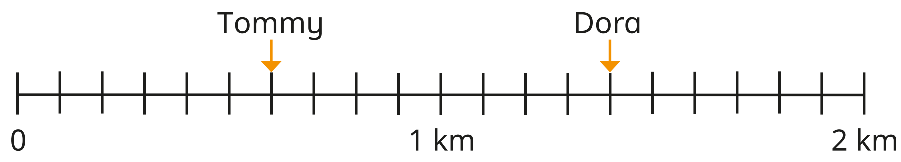
multiple possible answers, e.g.

$\frac{7}{8}$ to $1\frac{1}{8}$

$1\frac{1}{4}$ to $1\frac{1}{2}$

Dora and Tommy are completing a 2 km race.

The number line shows how far they have run so far.



How much further has Dora run than Tommy?

How much further do they each need to run?

Write your answers in their simplest form.

Huan has completed $\frac{1350}{2000}$ of the race.

Label the number line to show how far Huan has run so far.

$\frac{4}{5}$ km / Dora: $\frac{3}{5}$ km Tommy: $1\frac{2}{5}$ km

halfway between $1\frac{3}{10}$ and $1\frac{4}{10}$

Compare and order (denominator)

Notes and guidance

In this small step, children compare and order fractions with the same denominator. Building on the skills covered in the previous steps, they first need to use their knowledge of equivalent fractions to find a common denominator in order to compare.

Children begin by using bar models to help compare fractions. They first work with pairs of fractions where one denominator is a multiple of the other, building on learning from Year 5. They then look at pairs of fractions where the denominators are not multiples of each other, using their knowledge of multiples and common multiples. Encourage children to find the first common multiple, but allow them to explore different methods. Once children are confident expressing fractions with a common denominator, they use this to order fractions.

Things to look out for

- Some children may compare the numerators without looking at the denominators and finding equivalent fractions.
- Children may not always find the most efficient common multiple when multiplying the denominators, for example expressing $\frac{1}{6}$ and $\frac{2}{9}$ as $\frac{9}{54}$ and $\frac{12}{54}$ rather than $\frac{3}{18}$ and $\frac{4}{18}$

Key questions

- How could you use a number line or a bar model to help you compare the fractions?
- If the denominators are the same, how do you compare the fractions?
- Is one denominator a multiple of the other?
- If one denominator is not a multiple of the other, what do you need to do to be able to compare the fractions?
- How is comparing mixed numbers different from comparing proper fractions? How is it similar?

Possible sentence stems

- I am comparing ______ and ______. I can use ______ as the common denominator.
- If one denominator is not a multiple of the other, I need to find a ______

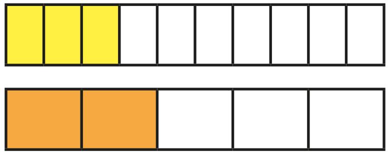
National Curriculum links

- Compare and order fractions, including fractions > 1
- Use common factors to simplify fractions; use common multiples to express fractions in the same denomination

Compare and order (denominator)

Key learning

- The bar models show $\frac{3}{10}$ and $\frac{2}{5}$



Which fraction is greater? How do you know?

- Alex is comparing $\frac{1}{5}$ and $\frac{4}{15}$

She uses equivalent fractions to help.

$\frac{1}{5} = \frac{3}{15}$ $\quad$ $\frac{3}{15} < \frac{4}{15}$ so $\frac{1}{5} < \frac{4}{15}$

Use Alex's method to compare the fractions.

$\frac{3}{20}$ and $\frac{1}{10}$ $\qquad$ $\frac{3}{4}$ and $\frac{20}{36}$ $\qquad$ $1\frac{2}{5}$ and $1\frac{10}{30}$

- Use the bar models to compare $\frac{3}{4}$ and $\frac{2}{5}$

$\frac{3}{4} \bigcirc \frac{2}{5}$

- Aisha is comparing $\frac{5}{6}$ and $\frac{3}{4}$ by finding the first common multiple of the denominators.

$\frac{5}{6} = \frac{10}{12}$ $\quad$ $\frac{3}{4} = \frac{9}{12}$

$\frac{10}{12} > \frac{9}{12}$ so $\frac{5}{6} > \frac{3}{4}$

Use Aisha's method to compare the fractions.

$\frac{4}{5} \bigcirc \frac{3}{4}$ $\qquad$ $\frac{3}{5} \bigcirc \frac{4}{7}$

$\frac{3}{4} \bigcirc \frac{7}{10}$ $\qquad$ $2\frac{2}{5} \bigcirc 2\frac{3}{8}$

- Write the fractions in descending order.

$\frac{1}{2}$ $\quad$ $\frac{1}{4}$ $\quad$ $\frac{7}{12}$ $\quad$ $\frac{3}{4}$ $\quad$ $\frac{1}{12}$ $\quad$ $\frac{11}{12}$

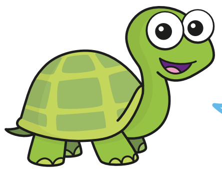
- Write the fractions in ascending order.

$\frac{3}{8}$ $\quad$ $\frac{11}{20}$ $\quad$ $\frac{1}{2}$ $\quad$ $\frac{2}{5}$ $\quad$ $\frac{3}{4}$ $\quad$ $\frac{7}{10}$

Compare and order (denominator)

Reasoning and problem solving

Tiny compares $\frac{3}{5}$ and $\frac{7}{15}$



$\frac{7}{15}$ is greater because 7 is greater than 3

Is Tiny correct?

Explain your answer.

No

Use the digit cards to complete the statements.

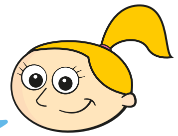
5 **3**

$\frac{\square}{4} > \frac{\square}{6}$ $\qquad$ $\frac{\square}{4} < \frac{6}{\square}$

$\frac{5}{4} > \frac{3}{6}$

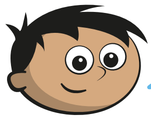
$\frac{3}{4} < \frac{6}{5}$ or $\frac{5}{4} < \frac{6}{3}$

Eva, Teddy and Amir are reading the same book.



I read $7\frac{3}{4}$ pages in 10 minutes.

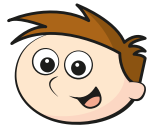
Eva



Amir

I read $\frac{15}{2}$ pages in 10 minutes.

I read $3\frac{1}{2}$ pages in 5 minutes.



Teddy

Who is reading the fastest?

How do you know?

Eva

Compare and order (numerator)

Notes and guidance

In the previous small step, children compared and ordered fractions using a common denominator. They now compare and order fractions with the same numerator.

Bar models are a useful representation to explore fractions with the same numerator, starting with unit fractions and then moving on to non-unit fractions. This will lead to the understanding that if the numerators are the same, then the greater the denominator, the smaller the fraction.

Children could visualise or place fractions on a number line and think about whether it is greater than or less than $\frac{1}{2}$ or if it is close to 0 or 1. Understanding can then be built on to compare fractions greater than 1

Children should consider whether it is more efficient to find a common numerator or a common denominator.

Things to look out for

- $\frac{1}{4}$ may be seen as smaller than $\frac{1}{5}$ because 4 is less than 5
- Children may need to be encouraged to use their knowledge of 0, 1 and $\frac{1}{2}$ to help compare fractions, for example $\frac{6}{10} > \frac{2}{7}$ because $\frac{6}{10} > \frac{1}{2}$ and $\frac{2}{7} < \frac{1}{2}$

Key questions

- How can you compare the fractions shown in the bar model?
- Do you need to change one or both numerators? Why?
- Is this fraction closer to 0 or 1?
- Is this fraction greater or less than $\frac{1}{2}$?
- Is it more efficient to find a common numerator or a common denominator?

Possible sentence stems

- When the numerators are the same, the ______ the denominator, the ______ the fraction.
- I know ______ is greater than $\frac{1}{2}$ because …
- I know ______ is closer to 1 than ______ because …

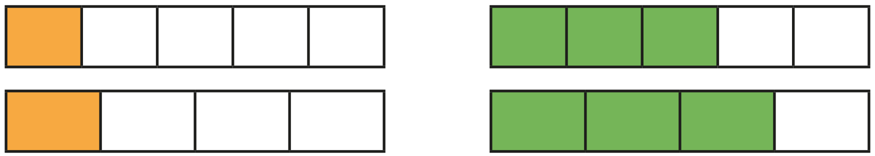
National Curriculum links

- Compare and order fractions, including fractions > 1

Compare and order (numerator)

Key learning

- Write <, > or = to compare the fractions.



$\frac{1}{5} \bigcirc \frac{1}{4}$ $\qquad$ $\frac{3}{5} \bigcirc \frac{3}{4}$

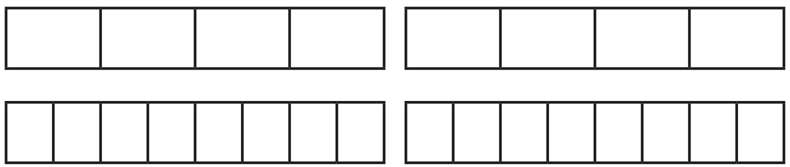
Complete the sentence.

When the numerators are the same, the ______ the denominator, the ______ the fraction.

- Write <, > or = to compare the fractions.

$\frac{4}{7} \bigcirc \frac{4}{5}$ $\qquad$ $\frac{3}{10} \bigcirc \frac{3}{11}$

- Complete the bar models and write <, > or = to compare the fractions.



$1\frac{3}{4} \bigcirc 1\frac{3}{8}$

- Whitney is comparing $\frac{2}{5}$ and $\frac{6}{13}$ using a common numerator.

$\frac{2}{5} = \frac{6}{15}$ $\qquad$ $\frac{6}{15} < \frac{6}{13}$ so $\frac{2}{5} < \frac{6}{13}$

Use Whitney's method to compare the numbers.

$\frac{3}{5} \bigcirc \frac{12}{17}$ $\qquad$ $\frac{6}{11} \bigcirc \frac{3}{5}$ $\qquad$ $1\frac{4}{7} \bigcirc 1\frac{8}{13}$

$\frac{5}{9} \bigcirc \frac{4}{7}$ $\qquad$ $\frac{8}{5} \bigcirc \frac{12}{7}$ $\qquad$ $7\frac{8}{9} \bigcirc 7\frac{12}{13}$

- Dani and Tom have completed a quiz.

Dani answered 7 out of 12 of her questions correctly.

Tom answered 21 out of 30 of his questions correctly.

Who got a greater fraction of their questions correct?

- Write each set of fractions in ascending order.

$\frac{3}{3}, \frac{3}{8}, \frac{3}{11}, \frac{3}{100}, \frac{3}{5}, \frac{3}{2}$ $\qquad$ $\frac{2}{8}, \frac{4}{7}, \frac{2}{10}, \frac{8}{12}, \frac{4}{6}$ $\qquad$ $2\frac{3}{7}, 2\frac{6}{9}, 2\frac{18}{20}, \frac{20}{7}, 2\frac{3}{10}$

Compare and order (numerator)

Reasoning and problem solving

Brett is comparing $\frac{3}{7}$ and $\frac{6}{11}$

How many different ways can he work this out?

Find a pair of fractions where it would be more efficient to find:

- a common numerator
- a common denominator.

Compare answers with a partner.

multiple possible answers, e.g.

common numerator:

$\frac{3}{7} = \frac{6}{14}$, $\frac{6}{14} < \frac{6}{11}$

so $\frac{3}{7} < \frac{6}{11}$

What could the missing number be, to make the statement true?

$$\frac{1}{5} > \frac{1}{\square} > \frac{1}{12}$$

Is there more than one answer?

How do you know?

6, 7, 8, 9, 10 or 11

Two different pieces of wood have had a fraction of their length chopped off.

Here are the pieces now, showing the fraction that is left.

A  $\frac{5}{6}$

B  $\frac{3}{4}$

Which piece of wood was longer to begin with?

Explain your answer.

The second piece of wood was 1 m long before it was cut.

How long was the first piece of wood?

B

90 cm

Add and subtract simple fractions

Notes and guidance

Before beginning, it may be appropriate to revise adding and subtracting fractions with the same denominator to remind children that where the denominators are the same, they need to add/subtract the numerators and leave the denominator unchanged. In this small step, children build on previous learning in this block and Year 5 to use equivalent fractions to add and subtract fractions where one denominator is a multiple of the other.

Children may be familiar with some common additions and subtractions such as $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ and this is a good example on which to build. They start by using bar models before moving on to finding the first common multiple of the denominators.

As the focus is on addition and subtraction of simple fractions, children are not yet required to work with improper fractions and mixed numbers as this will be looked at later in the block.

Things to look out for

- Children may not realise the need to make the denominators equal before adding.
- Children may add both the numerators and the denominators, for example $\frac{1}{2} + \frac{1}{4} = \frac{2}{6}$
- Children may not always simplify their answers.

Key questions

- Do the fractions have the same denominator?
- When are two fractions equivalent?
- How can you find a common denominator?
- How many of the fractions do you need to convert?
- Now the denominators are the same, how do you add/subtract the fractions?

Possible sentence stems

- Fractions must have the same ______ before they can be added or subtracted.
- The denominator has been multiplied by ______, so to make the equivalent fraction, multiply the numerator by ______
- When fractions have the same ______, to add or subtract them I just ______ the ______

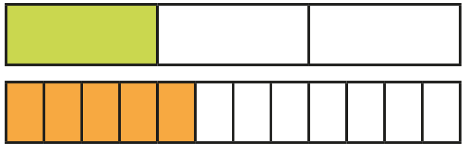
National Curriculum links

- Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions

Add and subtract simple fractions

Key learning

- Use the bar model to help add the fractions.

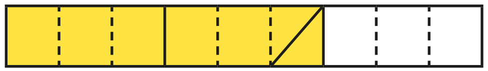


$\frac{1}{3} + \frac{5}{12}$

Work out the additions.

▶ $\frac{1}{3} + \frac{1}{12}$ ▶ $\frac{1}{3} + \frac{7}{12}$ ▶ $\frac{2}{3} + \frac{1}{12}$

- Use the bar model to work out the subtraction.



$\frac{2}{3} - \frac{1}{9}$

Work out the subtractions.

▶ $\frac{2}{3} - \frac{2}{9}$ ▶ $\frac{1}{3} - \frac{2}{9}$ ▶ $\frac{2}{3} - \frac{5}{9}$

- Here is a method for working out $\frac{7}{10} + \frac{7}{30}$

$$\frac{7}{10} = \frac{21}{30} \qquad \frac{21}{30} + \frac{7}{30} = \frac{28}{30} = \frac{14}{15}$$

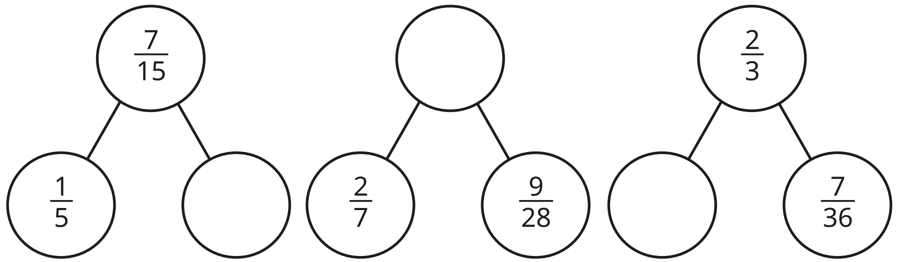
Use this method to work out the additions.

▶ $\frac{2}{9} + \frac{7}{27}$ ▶ $\frac{8}{15} + \frac{1}{5}$ ▶ $\frac{3}{16} + \frac{3}{8} + \frac{1}{4}$

- Find the difference between each pair of fractions.

$\frac{3}{4}$ and $\frac{5}{8}$	$\frac{7}{12}$ and $\frac{1}{3}$	$\frac{14}{15}$ and $\frac{2}{5}$	$\frac{8}{9}$ and $\frac{1}{3}$

- Complete the part-whole models.

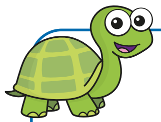


- Ms Lee has a full tin of paint.
  - She uses $\frac{1}{5}$ of the paint on Monday.
  - She uses $\frac{1}{20}$ on Tuesday.
  - She uses $\frac{3}{10}$ on Wednesday.

How much paint does she have left?

Add and subtract simple fractions

Reasoning and problem solving



Tiny is adding fractions.

Here are Tiny's workings.

$$\frac{3}{5} + \frac{1}{15} = \frac{4}{20} = \frac{1}{5}$$

Explain Tiny's mistake.

Find the correct answer.

$\frac{10}{15} = \frac{2}{3}$

Use the same digit in both boxes to complete the calculation.

$$\frac{\square}{20} + \frac{1}{\square} = \frac{9}{20}$$

Find all the possible answers.

$\frac{4}{20} + \frac{1}{4} = \frac{9}{20}$

$\frac{5}{20} + \frac{1}{5} = \frac{9}{20}$

Find the missing number.

$$\frac{3}{5} + \frac{1}{20} = \frac{3}{4} - \frac{\square}{10}$$

1

Kim subtracts $\frac{3}{5}$ from a fraction.

What fraction has Kim subtracted $\frac{3}{5}$ from?

Give your answer in its simplest form.

$\frac{7}{9}$

Add and subtract any two fractions

Notes and guidance

Following on from the previous small step, children add and subtract fractions where the denominators are not multiples of each other.

Children may need to revisit how to find a common denominator before completing the calculations. They use bar models and then move on to finding the first common multiple of the denominators. Once this is secure, they add up to three fractions or subtract fractions with different denominators.

Children add fractions with answers greater than one, but do not add and subtract mixed numbers until the next step.

Encourage children to simplify answers and convert improper fractions to mixed numbers as appropriate.

Things to look out for

- Children may add both the numerators and the denominators, for example $\frac{1}{3} + \frac{1}{4} = \frac{2}{7}$
- Children may not always simplify their answers.
- Children may leave answers as improper fractions, for example $\frac{7}{5}$

Key questions

- Do the fractions have the same denominator?
- What is the first common multiple of ______ and ______?
- How many of the fractions do you need to convert?
- How do you know if your answer is in its simplest form?
- Do you need to convert your answer to a mixed number? Why or why not?

Possible sentence stems

- The lowest common multiple of ______ and ______ is ______
- To add/subtract the fractions, I could convert them both to ______
- When fractions have the same ______, to add or subtract them you just ______ the ______

National Curriculum links

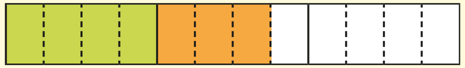
- Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- Identify common factors, common multiples and prime numbers

Add and subtract any two fractions

Key learning

- Esther is working out $\frac{1}{3} + \frac{1}{4}$

  She finds a common denominator to work out the answer.

  $\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$

  Use Esther's method to work out the additions.

  $\frac{1}{4} + \frac{2}{3}$ $\quad$ $\frac{1}{4} + \frac{1}{5}$ $\quad$ $\frac{1}{4} + \frac{3}{5}$

- What common denominator would you use to add each pair of fractions?

  $\frac{2}{4}$ and $\frac{1}{5}$ $\quad$ $\frac{1}{6}$ and $\frac{2}{5}$ $\quad$ $\frac{1}{3}$ and $\frac{5}{7}$ $\quad$ $\frac{3}{8}$ and $\frac{4}{7}$

  Find the sum of each pair.

- On Friday, Scott walked $\frac{5}{6}$ km to school, then $\frac{3}{4}$ km to the shop and then $\frac{4}{5}$ km home.

  How far did he walk altogether?

- Annie is calculating $\frac{7}{9} - \frac{1}{2}$

  She finds the first common multiple of 9 and 2

  first common multiple of 9 and 2 is 18 $\frac{7}{9} - \frac{1}{2} = \frac{14}{18} - \frac{9}{18} = \frac{5}{18}$

  Use this method to find the differences.

  $\frac{2}{3} - \frac{1}{5}$ $\quad$ $\frac{4}{9} - \frac{1}{6}$ $\quad$ $\frac{5}{7} - \frac{1}{3}$ $\quad$ $\frac{11}{12} - \frac{3}{8}$

- Kim has $\frac{3}{4}$ kg of carrots and $\frac{2}{5}$ kg of potatoes.

  She is calculating the total mass of the carrots and potatoes.

  $\frac{3}{4} + \frac{2}{5} = \frac{15}{20} + \frac{8}{20} = \frac{23}{20} = 1\frac{3}{20}$ kg

  Use Kim's method to find the sums.

  Give your answers as mixed numbers.

  $\frac{3}{4} + \frac{3}{5}$ $\quad$ $\frac{7}{8} + \frac{1}{3}$ $\quad$ $\frac{5}{6} + \frac{5}{7}$ $\quad$ $\frac{13}{20} + \frac{2}{3}$

- Write <, > or = to complete the statements.

  $\frac{1}{3} + \frac{1}{5}$ ◯ $\frac{4}{5} - \frac{1}{3}$ $\qquad$ $\frac{1}{3} - \frac{1}{5}$ ◯ $\frac{4}{5} - \frac{1}{3}$

Add and subtract any two fractions

Reasoning and problem solving

Huan and Dora are working out $\frac{1}{4} + \frac{5}{6}$

Here are their methods.

Huan

$$\frac{1}{4} + \frac{5}{6} = \frac{6}{24} + \frac{20}{24} = \frac{26}{24} = 1\frac{2}{24}$$

Dora

$$\frac{1}{4} + \frac{5}{6} = \frac{3}{12} + \frac{10}{12} = \frac{13}{12} = 1\frac{1}{12}$$

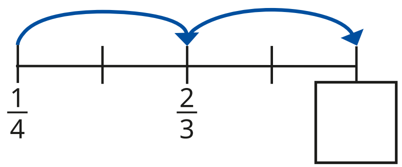
Who is correct?

Explain your answer.

Both are correct.

The jumps on the number line are equal.

What is the missing value on the number line?



$1\frac{1}{12}$

Fill in the boxes to make the calculation correct.

$$1\frac{\square}{10} = \frac{4}{\square} + \frac{\square}{10}$$

multiple possible answers, e.g.

$1\frac{1}{10} = \frac{4}{5} + \frac{3}{10}$

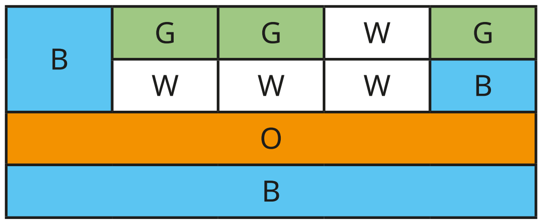
$1\frac{7}{10} = \frac{4}{5} + \frac{9}{10}$

A wall has been painted in different colours.

$\frac{1}{4}$ of the wall is orange (O).

What fraction of the wall is blue (B)?

What fraction of the wall is white (W)?



blue = $\frac{2}{5}$

white = $\frac{1}{5}$

Add mixed numbers

Notes and guidance

Children encountered mixed numbers in the answers to additions in the previous small step. They now add two mixed numbers, building on their experience of this in Year 5

Children explore adding the wholes and fractional parts separately. This is usually the most efficient method, but converting to improper fractions and then adding is an alternative. Some children may need to revisit converting between improper fractions and mixed numbers.

Questions begin with fractions with the same denominator and then move on to fractions with different denominators. Children can still draw models to represent adding fractions, particularly if these are useful for pairs of fractions with unequal denominators.

Things to look out for

- Children may make errors in the partitioning or recombining of the integer and fractional parts.
- Children may make arithmetical errors when converting to improper fractions with larger numbers.
- The cognitive load is significant when finding solutions to these multi-step problems, so providing scaffolding/partially started solutions may be useful.

Key questions

- How can you partition the mixed numbers?
- How can the addition be rewritten to make it easier?
- In this question, is it easier to deal with wholes and fractions or to use improper fractions? Why?
- How do you convert a mixed number into an improper fraction?
- Are there any improper fractions in the answer? What can you do about this?

Possible sentence stems

- Mixed numbers can be partitioned into a ______ part and a ______ part.
- A fraction is improper when the ______ is greater than the ______
- ______ is made up of ______ wholes and ______

National Curriculum links

- Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- Identify common factors, common multiples and prime numbers

Add mixed numbers

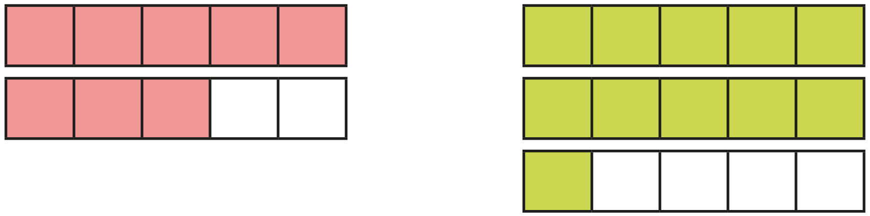
Key learning

- What method would you use to work out the additions?

$3\frac{2}{7} + 4$	$3\frac{2}{7} + \frac{4}{7}$	$3\frac{2}{7} + 4\frac{4}{7}$

How are they similar? How are they different?

- Aisha uses a bar model to help work out $1\frac{3}{5} + 2\frac{1}{5} = 3\frac{4}{5}$



Use bar models to help work out the additions.

$1\frac{2}{7} + 3\frac{2}{7}$	$3\frac{2}{7} + 1\frac{4}{7}$	$2\frac{1}{7} + 3\frac{5}{7}$	$2\frac{1}{7} + 3\frac{6}{7}$

- Work out the total of each pair of fractions.

$\frac{3}{11} + \frac{2}{11}$	$1\frac{3}{11} + \frac{2}{11}$	$1\frac{3}{11} + 1\frac{2}{11}$	$2\frac{3}{11} + 1\frac{2}{11}$

How did you work them out?

Compare methods with a partner.

- Rosie and Amir are working out $1\frac{1}{2} + 2\frac{1}{6}$

Rosie

$$1 + 2 = 3$$

$$\frac{1}{2} + \frac{1}{6} = \frac{3}{6} + \frac{1}{6} = \frac{4}{6}$$

$$3 + \frac{4}{6} = 3\frac{4}{6} = 3\frac{2}{3}$$

Amir

$$1\frac{1}{2} + 2\frac{1}{6} = \frac{3}{2} + \frac{13}{6}$$

$$= \frac{9}{6} + \frac{13}{6}$$

$$= \frac{22}{6} = 3\frac{4}{6} = 3\frac{2}{3}$$

Whose method do you prefer?

Explain your answer.

Use your preferred method to add the mixed numbers.

$3\frac{1}{2} + 2\frac{3}{8}$	$2\frac{1}{9} + 2\frac{2}{5}$	$34\frac{1}{9} + 5\frac{5}{8}$	$2\frac{3}{4} + 1\frac{4}{5}$

- A jug contains $2\frac{3}{4}$ litres of juice.

  Another jug contains $3\frac{3}{5}$ litres of juice.

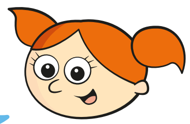
  How much juice is there altogether?

Add mixed numbers

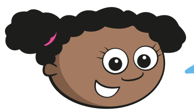
Reasoning and problem solving

Alex, Whitney and Teddy are trying to run 10 km between them.

I ran $2\frac{1}{2}$ km.



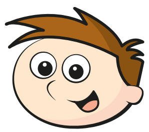
Alex



Whitney

I ran $3\frac{4}{5}$ km.

I ran $3\frac{6}{10}$ km.



Teddy

How far have they run?

How much further do they need to run?

$9\frac{9}{10}$ km

$\frac{1}{10}$ km

On Saturday and Sunday, Nijah ran a total of $4\frac{1}{2}$ km.

Suggest how far Nijah ran on each day.

Find more than one answer.

multiple possible answers, e.g.

$2\frac{1}{3}$ km and $2\frac{1}{6}$ km

The numbers in the row and column add up to make the totals shown.

$2\frac{1}{4}$	$\square\frac{\square}{8}$	$\frac{1}{2}$	$= 3\frac{7}{8}$
$\frac{1}{\square}$			
$3\frac{1}{12}$			
$= 5\frac{1}{2}$			

Find the missing values.

$1\frac{1}{8}$

$\frac{1}{6}$

Subtract mixed numbers

Notes and guidance

In this small step, children subtract two mixed numbers, building on the learning from Year 5. Children make links between what is the same and what is different when subtracting mixed numbers compared to adding them.

To introduce this step, children subtract mixed numbers that have the same denominator and do not break the whole. They then subtract fractions with different denominators and complete questions that break the whole. When breaking the whole, children can exchange one whole or convert mixed numbers to improper fractions. Bar models are useful tools to illustrate both methods, and number lines can be used to help find the difference.

Things to look out for

- When breaking the whole, children may be unsure how to exchange.
- Children may make errors when partitioning mixed numbers, for example they may not correctly convert $3\frac{3}{4}$ to $2\frac{7}{4}$
- Children should think about which method is most appropriate for the question, rather than relying on just one method.

Key questions

- How can you partition the mixed number?
- How can the subtraction be rewritten to make it easier?
- In this question, is it easier to deal with wholes and fractions or to use improper fractions? Why?
- How do you convert a mixed number into an improper fraction?

Possible sentence stems

- This calculation will/will not cross the whole because …
- A fraction is equal to one whole when the ______ is equal to the ______
- The mixed number can be partitioned into ______ and ______
- ______ can be written as ______ wholes and ______

National Curriculum links

- Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- Identify common factors, common multiples and prime numbers

Subtract mixed numbers

Key learning

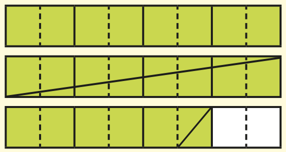
- What method would you use to work out the subtractions?

$3\frac{7}{8} - 1$	$3\frac{7}{8} - \frac{3}{8}$	$3\frac{7}{8} - 1\frac{3}{8}$

  Compare methods with a partner.

  How is this similar to addition? How is it different?

- Tom uses bar models to help work out $2\frac{3}{4} - 1\frac{3}{8}$

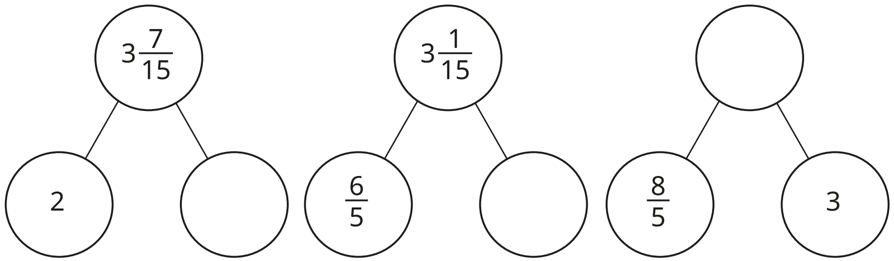


$2\frac{3}{4} - 1\frac{1}{8} = 1 + \frac{5}{8} = 1\frac{5}{8}$

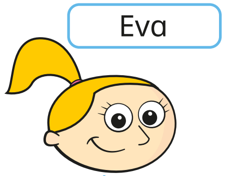
  Use bar models to help work out the subtractions.

$2\frac{3}{4} - 1\frac{5}{8}$	$3\frac{3}{4} - 2\frac{3}{8}$	$2\frac{1}{2} - 1\frac{3}{10}$	$4\frac{1}{3} - 2\frac{1}{3}$

  Complete the part-whole models.



- Eva and Tommy are working out $3\frac{3}{5} - 1\frac{7}{10}$



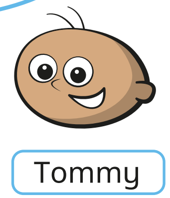
$3\frac{3}{5} = 2\frac{8}{5}$

$3\frac{3}{5} - 1\frac{7}{10} = 2\frac{8}{5} - 1\frac{7}{10} = 2\frac{16}{10} - 1\frac{7}{10} = 1\frac{9}{10}$

I cannot subtract the wholes and fractions separately because $\frac{3}{5}$ is less than $\frac{7}{10}$ I will exchange 1 whole for 5 fifths.

I will convert both mixed numbers to improper fractions and then subtract them.

$3\frac{3}{5} - 1\frac{7}{10} = \frac{18}{5} - \frac{17}{10} = \frac{36}{10} - \frac{17}{10} = \frac{19}{10} = 1\frac{9}{10}$

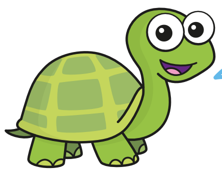


  Choose a method to work out the subtractions.

$4\frac{4}{5} - 1\frac{9}{10}$	$2\frac{1}{7} - 1\frac{1}{3}$	$3\frac{5}{12} - 1\frac{7}{9}$	$3\frac{5}{11} - 1\frac{4}{5}$

Subtract mixed numbers

Reasoning and problem solving



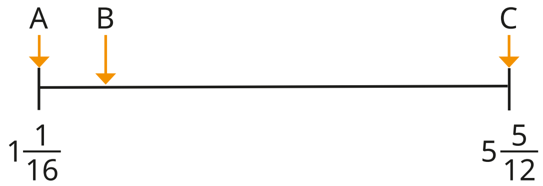
I cannot work out $3\frac{1}{3} - 1\frac{1}{2}$ because $\frac{1}{2}$ is greater than $\frac{1}{3}$

Is Tiny correct?

Explain your answer.

No

On the number line, C is $3\frac{2}{3}$ more than B.



What is the value of B?

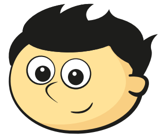
What is the difference between A and B?

$1\frac{3}{4}$

$\frac{11}{16}$

Jack is calculating $4\frac{2}{7} - 2\frac{6}{7}$

He adds $\frac{1}{7}$ to both numbers.

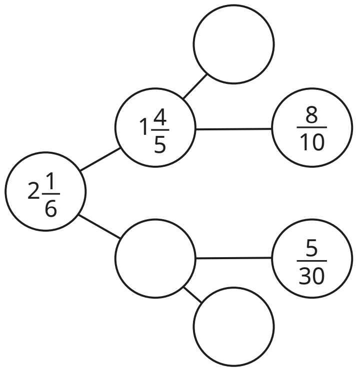


$4\frac{2}{7} - 2\frac{6}{7} = 4\frac{3}{7} - 3$, so the answer is $1\frac{3}{7}$

Explain why Jack is correct.

He increased both numbers by $\frac{1}{7}$ so the difference remained constant.

Complete the part-whole model.



1

$\frac{11}{30}$

$\frac{1}{5}$

Multi-step problems

Notes and guidance

In this small step, children apply the skills they have learnt in previous steps to solving problems in real-life contexts.

The problems may involve more than one calculation and children need to choose the operations and consider what order to perform them in; this will need careful modelling. Encourage children to think about the most appropriate method to perform any of the calculations. Sharing methods could help children gain a flexible approach to solving the problems.

Children also need to ensure that they write fractions in their simplest form and convert between improper fractions and mixed numbers where appropriate.

Things to look out for

- For longer word problems, the questions may need to be broken down into separate sections to scaffold learning.
- If their understanding is not secure, children may need to revise earlier learning before completing the problems.
- Children may need support to set out solutions with several parts clearly.
- Some children may struggle with the maths because they are overwhelmed by the context of a question.

Key questions

- What can you work out first?
- What do you need to know to work out the answer?
- Can you draw a diagram to represent the problem?
- Can you work out the answer to this part of the problem mentally or do you need another method?
- What can you do next?

Possible sentence stems

- First, I need to work out …
- The calculation I need to do is …
- Next, I need to work out …

National Curriculum links

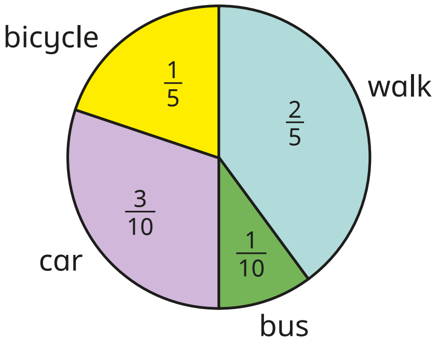
- Use common factors to simplify fractions; use common multiples to express fractions in the same denomination
- Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- Solve problems involving addition, subtraction, multiplication and division

Multi-step problems

Key learning

- Children in Class 6 were asked how they travel to school.

  The results of the survey are shown in the pie chart.



  What fraction of children do not get the bus to school?

- Dr Fisher has $\frac{7}{8}$ of a tank of petrol in his car.

  He drives to see his friend and uses $\frac{1}{5}$ of a tank of petrol.

  What fraction of a tank of petrol is left in the tank?

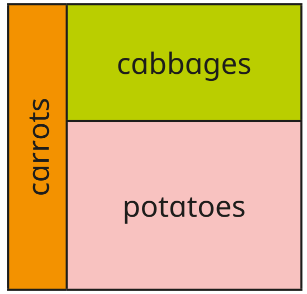
- A family buys two equal-sized boxes of cereal.

  In one week, they eat $\frac{2}{3}$ of box A and $\frac{3}{5}$ of box B.

  How many boxes of cereal do they eat that week?

  How many boxes of cereal will they need for three weeks?

- Here is a vegetable patch.



  $\frac{1}{5}$ of the patch is for carrots

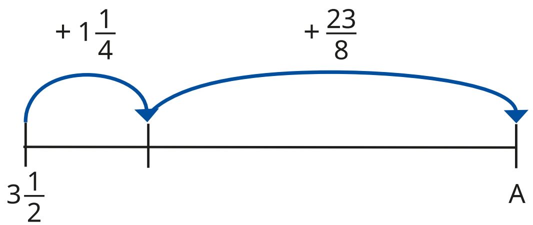
  and $\frac{3}{8}$ of the patch is for cabbages.

  What fraction of the patch is for potatoes?

  How much more of the patch is for the potatoes than for the cabbages?

  Give all your answers in their simplest form.

- What is the value of A?



- Whitney has 5 bags of raisins.

  On Monday, she eats $\frac{2}{3}$ of a bag and gives $\frac{4}{5}$ of a bag away.

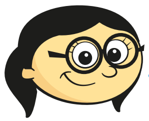
  On Tuesday, she eats $1\frac{1}{3}$ bags and gives $\frac{2}{3}$ of a bag away.

  How many bags of raisins does Whitney have left?

Multi-step problems

Reasoning and problem solving

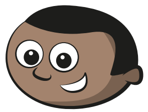
Annie and Mo are going on a trip.



Annie

My suitcase has a mass of $29\frac{1}{2}$ kg.

My suitcase is $2\frac{1}{5}$ kg lighter than Annie's.



Mo

What is the total mass of the suitcases?

There is a weight allowance of 32 kg per suitcase.

How much below the weight allowance are Annie and Mo's suitcases?

$56\frac{4}{5}$ kg

Annie: $2\frac{1}{2}$ kg

Mo: $4\frac{7}{10}$ kg

Find the value of ☀

$$♥ + 3\frac{4}{9} = 6\frac{1}{3}$$

$$8\frac{1}{10} - ♥ = ☀$$

$☀ = 5\frac{19}{90}$

Complete the calculation.

$$2\frac{9}{12} + 3\frac{15}{20} - 2\frac{3}{4} - 2\frac{75}{100} = \boxed{}$$

How can you make this calculation simpler?

1