

Autumn Block 4

**Multiplication
and division A**

Small steps

Step 1

Multiples of 3

Step 2

Multiply and divide by 6

Step 3

6 times-table and division facts

Step 4

Multiply and divide by 9

Step 5

9 times-table and division facts

Step 6

The 3, 6 and 9 times-tables

Step 7

Multiply and divide by 7

Step 8

7 times-table and division facts

Small steps

Step 9

11 times-table and division facts

Step 10

12 times-table and division facts

Step 11

Multiply by 1 and 0

Step 12

Divide a number by 1 and itself

Step 13

Multiply three numbers

Multiples of 3

Notes and guidance

This small step revisits learning from Year 3 around multiplying by 3 and the 3 times-table.

Children explore the link between counting in 3s and the 3 times-table to understand multiples of 3 in a range of contexts. They use familiar representations such as number tracks and hundred squares to represent multiples of 3. They explore how to recognise if a number is a multiple of 3 by finding its digit sum: if the sum of the digits of a number is a multiple of 3, then the number itself is also a multiple of 3.

This small step includes multiples of 3 up to 3×12 and will be useful support for learning multiples of 6 and 9 in future steps.

Things to look out for

- Children may think that any number with 3 ones is a multiple of 3
- An early mistake when counting in 3s will affect all subsequent multiples.
- Children may always begin counting from 3 to find a larger multiple of 3, when they could use the multiples they already know to find the new information.

Key questions

- What is the next multiple of 3?
- What is the multiple of 3 before _____?
- How many 3s are there in _____?
- How do you find the digit sum of a number?
- How can you tell if a number is a multiple of 3?
- Are the multiples of 3 odd or even?

Possible sentence stems

- The next multiple of 3 is _____
- The multiple of 3 before _____ is _____
- I know _____ is a multiple of 3 because ...

National Curriculum links

- Recall multiplication and division facts for multiplication tables up to 12×12
- Recognise and use factor pairs and commutativity in mental calculations

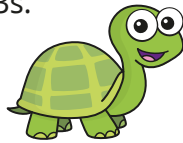
Multiples of 3

Key learning

- Complete the number track.

3	6		12		18	21	24			33	36
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- Tiny is counting in 3s.



3, 6, 9, 13, 16,
19, 23 ...

What mistake has Tiny made?

- Colour the multiples of 3 in the hundred square.

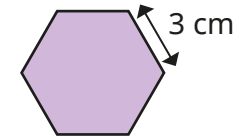
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

What do you notice?

- Complete the statements.

- ▶ 3 lots of 3 = _____
- ▶ 4 lots of 3 = _____
- ▶ 5 lots of 3 = _____
- ▶ 10 lots of 3 = _____
- ▶ 4 lots of 3 and 2 lots of 3 = _____ lots of 3
- ▶ 7 lots of 3 = _____ lots of 3 and 5 lots of 3

- Each side of a regular hexagon measures 3 cm.



What is the perimeter of the shape?

- 3 cars each have 3 people inside.

Each person has 3 bags.

How many bags are there altogether?



Multiples of 3

Reasoning and problem solving

Here are some multiples of 3



462 717 897 612 900 561

Find the digit sum of each number.

What do you notice?



Use what you have learned about adding digits together to find which of the numbers are multiples of 3

471 418 393

297 156 206

12, 15, 24, 9, 9, 12

471, 393, 297, 156

Scott has 3 times as much money as Kim.



Kim has 3 times as much money as Amir.

Kim has £12

How much money do Scott and Amir each have?

Scott: £36

Amir: £4

Bags of sweets cost £3



- Ron buys 3 bags.
- Dani buys 9 bags.
- Aisha buys 4 bags.



How much does each person spend?

How much more does Dani spend than Aisha?

How much do the children spend in total?

Ron: £9

Dani: £27

Aisha: £12

£15 more

£48 altogether

Multiply and divide by 6

Notes and guidance

In this small step, children build on their knowledge of the 3 times-table to explore the 6 times-table. The step aims to embed the children's fluency skills with the 6 times-table, while also providing them with strategies to use the multiplication facts they know to find unknown facts.

Children explore the fact that the 6 times-table is double the 3 times-table. Children who are confident in their times-tables can also explore the link between the 5 and 6 times-tables. They use the fact that multiplication is commutative to derive values for the 6 times-tables. This is developed further with division facts, where children explore fact families to embed their understanding of division as the inverse of multiplication.

Things to look out for

- Children may always start at $1 \times 6 = 6$ and recite the times-table, rather than use the number facts they know to find the facts they are not as secure with.
- When writing fact families, children may follow the pattern from multiplication and see division as commutative, for example writing $42 \div 6 = 7$ so $6 \div 42 = 7$
- Children may not recognise that when they are dividing by a greater number they get a smaller answer.

Key questions

- How many equal groups do you have?
How many are there in each group?
How many are there altogether?
- What does each number in the calculation represent?
- What does commutative mean?
- Is multiplication/division commutative?
- How can you use facts from the 3 times-table to work out facts from the 6 times-table?

Possible sentence stems

- 6 lots of _____ is _____
- _____ shared into 6 equal groups is _____
- Multiplying by 6 is the same as multiplying by _____ twice.
- _____ \times 6 = double _____ \times 3

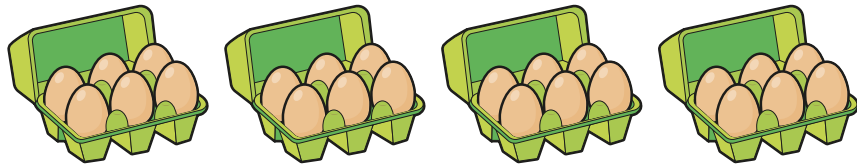
National Curriculum links

- Recall multiplication and division facts for multiplication tables up to 12×12
- Recognise and use factor pairs and commutativity in mental calculations

Multiply and divide by 6

Key learning

- Complete the sentences.

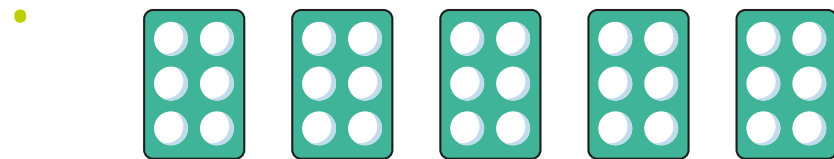


There are _____ boxes.

Each box contains _____ eggs.

There are _____ eggs in total.

_____ × _____ = _____



Complete the fact family.

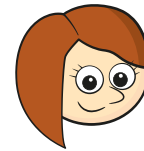
_____ × _____ = _____

_____ ÷ _____ = _____

_____ × _____ = _____

_____ ÷ _____ = _____

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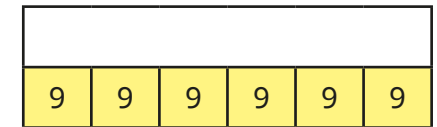
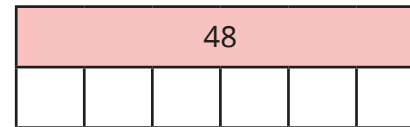


I can find the 6 times-table by doubling the 3 times-table.

Use Rosie's method to complete the sentences.

- ▶ $3 \times 6 = \text{double } 3 \times 3 = \text{double } 9 = 18$
- ▶ $4 \times 6 = \text{double } 4 \times \text{_____} = \text{_____} = \text{_____}$
- ▶ $5 \times 6 = \text{double } \text{_____} \times \text{_____} = \text{_____} = \text{_____}$
- ▶ $7 \times 6 = \text{double } \text{_____} \times \text{_____} = \text{_____} = \text{_____}$

- Complete the bar models.



Write the fact families for each bar model.

- Which numbers can be divided into equal groups of 6?

24

18

48

60

9

56

72

38

Multiply and divide by 6

Reasoning and problem solving

$6 \times 12 = 72$
 so $12 \div 6 = 72$

Is Tiny correct?
Explain your answer.

No

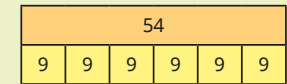
Dora puts 72 pencils into pots.
 She puts 6 pencils into each pot.
 She shares the pots equally
 between 6 tables.
 How many pots does she put on
 each table?

2

Draw a bar model to represent each problem.

Tom has 54 cakes.
He shares them equally into 6 boxes.
How many cakes will go in each box?

Tom puts 54 cakes into boxes.
There are 6 cakes in each box.
How many boxes will he need?



Is the statement true or false?

Sharing an amount into 6 equal groups will give twice as many in each group as sharing the same amount into 3 equal groups.

Explain your answer.



False

6 times-table and division facts

Notes and guidance

Building on the previous step, children use known facts to become more fluent in using the 6 times-table.

As in the previous step, they apply knowledge of the 3 times-table and understand that each multiple of 6 is double the corresponding multiple of 3

Children use their knowledge of other times-tables to find values for the 6 times-table, for example finding that $6 \times 7 = 42$ because $5 \times 7 = 35$ and $1 \times 7 = 7$, so $35 + 7 = 42$

It is important that children practise the related division facts as well as the multiplication facts associated with the 6 times-table. Fluency with the 6 times-table will also help children to work out the 12 times-table in future steps.

Things to look out for

- Children may confuse different terminology to describe multiplication and division such as “equal groups”, “lots of”, “times”, “multiple” and so on.
- An early mistake when counting in 6s will affect all subsequent multiples.
- Children may not see the link between $6 \times \underline{\quad}$ and other multiples such as $5 \times \underline{\quad}$ and $1 \times \underline{\quad}$

Key questions

- How can you use facts from the 3 times-table to work out facts in the 6 times-table?
- How can you use facts from the 5 times-table to work out facts in the 6 times-table?
- If you know a multiplication sentence, what division sentences can you find?
- What is the fact family for the calculation?

Possible sentence stems

- 6 multiplied by _____ is equal to _____
- _____ \times 6 = double _____ \times 3
- _____ \times 6 = _____ \times 5 + _____
- _____ \times 6 = _____, so _____ \div 6 = _____

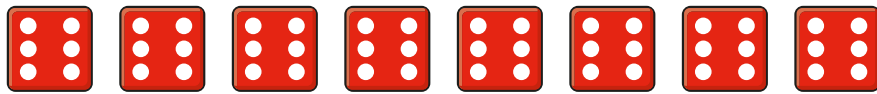
National Curriculum links

- Recall multiplication and division facts for multiplication tables up to 12×12
- Recognise and use factor pairs and commutativity in mental calculations

6 times-table and division facts

Key learning

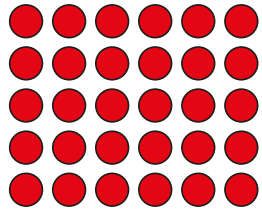
- Write a multiplication fact to work out the total.



_____ × _____ = _____

What other multiplication and division facts can you find?

- Complete the fact family to match the array.



_____ × _____ = _____

_____ × _____ = _____

_____ ÷ _____ = _____

_____ ÷ _____ = _____

- Complete the number sentences.

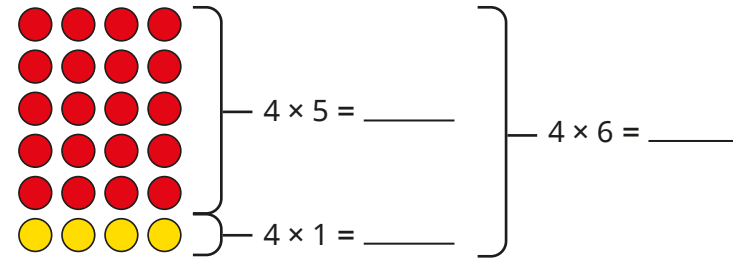
▶ $1 \times 3 = \underline{\quad}$ $1 \times \underline{\quad} = 6$

▶ $2 \times \underline{\quad} = 6$ $2 \times 6 = \underline{\quad}$

▶ $3 \times 3 = \underline{\quad}$ $3 \times 6 = \underline{\quad}$

Write the next two sentences in the pattern.

- Use the array to complete the number sentences.



Use this method to complete the number sentences.

▶ $2 \times 5 = \underline{\quad}$ and $2 \times 1 = \underline{\quad}$ so $2 \times 6 = \underline{\quad}$

▶ $3 \times 5 = \underline{\quad}$ and $3 \times 1 = \underline{\quad}$ so $3 \times 6 = \underline{\quad}$

▶ $7 \times 5 = \underline{\quad}$ and $7 \times 1 = \underline{\quad}$ so $7 \times 6 = \underline{\quad}$

- Match the inverse operations.

$7 \times 6 = 42$

$18 \div 6 = 3$

$3 \times 6 = 18$

$72 \div 6 = 12$

$9 \times 6 = 54$

$54 \div 6 = 9$


$12 \times 6 = 72$


$42 \div 6 = 7$

6 times-table and division facts

Reasoning and problem solving

Dexter is thinking of two numbers.






The sum of my numbers is 15 and their product is 54

6 and 9

What are Dexter's numbers?
Explain your answer.




Think of your own problem like this for a partner to solve.

Is the statement true or false?

All multiples of 3 are multiples of 6

False

Explain your answer.



Here are some facts about multiples of 3 and 6

If an even number has a digit sum that is a multiple of 3, then the number is a multiple of 3 and 6

If an odd number has a digit sum that is a multiple of 3, then it is a multiple of 3 but not of 6

195

15

624

592

128

348

Multiple of 3 only	Multiple of 3 and 6	Not a multiple of 3 or 6

multiple of 3 only: 195, 15

multiple of 3 and 6: 624, 348

not a multiple of 3 or 6: 592, 128

Multiply and divide by 9

Notes and guidance

In this small step, children are introduced to the 9 times-table. They use a range of strategies to support their fluency, such as looking for number patterns and finding unknown number facts from known facts, for example subtracting from the 10 times-table or tripling the 3 times-table, and these will be built upon later in the block.

Children explore the structure of the 9 times-table using a range of models and pictorial representations, and by exploring multiples of 9 in context. They also use commutativity with the facts they already know from other times-tables.

Children find division facts and explore fact families to embed their understanding of division as the inverse of multiplication.

Things to look out for

- When finding multiplication facts, children may always start at $1 \times 9 = 9$ and recite the times-table rather than using the number facts they know to find the facts they are not as secure with.
- When writing fact families, children may follow the pattern from multiplication and see division as commutative, writing examples such as $54 \div 9 = 6$ so $9 \div 54 = 6$

Key questions

- How many equal groups are there?
How many are there in each group?
How many are there altogether?
- How can you use the 10 times-table to work out the 9 times-table?
- How can you use the 3 times-table to work out the 9 times-table?
- What does each number in the calculation represent?
- What patterns can you see in the 9 times-table?

Possible sentence stems

- 9 lots of _____ is equal to _____
- _____ groups of _____ is equal to _____ groups of _____
- _____ \times 10 = _____, so _____ \times 9 = _____ - _____ = _____

National Curriculum links

- Recall multiplication and division facts for multiplication tables up to 12×12
- Recognise and use factor pairs and commutativity in mental calculations

Multiply and divide by 9

Key learning

- Complete the sentences to describe the oranges.

▶ There are _____ rows of 4 oranges.

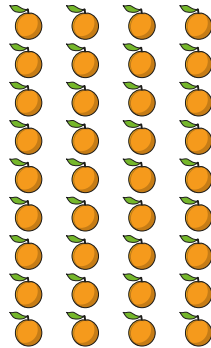
There are _____ oranges in total.

_____ × _____ = _____

▶ The oranges are shared into 9 boxes.

There are _____ oranges in each box.

_____ ÷ _____ = _____



- Complete the number track.



What do you notice?

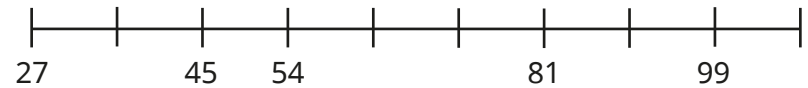
- Here are Annie's workings for 9×7

$$\begin{aligned}
 9 \times 7 &= 10 \times 7 - 7 \\
 &= 70 - 7 \\
 &= 63
 \end{aligned}$$

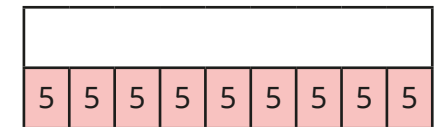
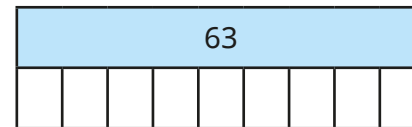
Use Annie's method to complete the number sentences.

- ▶ $9 \times 3 = 10 \times \underline{\quad} - \underline{\quad}$
- ▶ $9 \times 8 = 10 \times \underline{\quad} - \underline{\quad}$
- ▶ $9 \times 6 = 10 \times \underline{\quad} - \underline{\quad}$
- ▶ $9 \times 9 = 10 \times \underline{\quad} - \underline{\quad}$

- Complete the number line to show counting in multiples of 9



- Complete the bar models.



Write the fact families for each bar model.

- Mrs Trent has 36 boxes of pencils. She shares them equally between 9 classes. How many boxes of pencils does each class get?
- Tommy packs 72 eggs into boxes. Each box contains 9 eggs. How many boxes does he need?

Multiply and divide by 9

Reasoning and problem solving

Here are some multiples of 9



36 45 279 459 981 108

Find the digit sum of each number.

What do you notice?



Use what you have learnt about adding the digits together to find out which of these numbers are multiples of 9

477

418

393

999

396

576

9, 9, 18, 18, 18, 9

477, 999, 396, 576

Amir has 9 bags of 6 sweets.



Whitney has 6 bags of 9 sweets.



Amir

I have more sweets, because I have more bags.



I have more sweets, because I have more in each bag.



Whitney

Neither is correct.

Who is correct?

Explain your reasoning.



9 times-table and division facts

Notes and guidance

Building on the previous step, children become more fluent using the 9 times-table and apply the multiplication and division facts in a wide variety of contexts.

To establish the facts, children use strategies such as using the 10 times-table to derive the 9 times-table, and understanding that each multiple of 9 is triple the equivalent multiple of 3

They investigate finding the digit sum and look for patterns that will support them in identifying multiples of 9: if the sum of the digits of a number is a multiple of 9, then the number itself is also a multiple of 9. This, and the corresponding rule for the 3 times-table, will support their learning in the next step, where they compare the 3, 6 and 9 times-tables.

Things to look out for

- Children may confuse different terminology to describe multiplication and division, such as “equal groups”, “lots of”, “times”, “multiple” and so on.
- An early mistake when counting in 9s will affect all subsequent multiples.
- Children may use tricks to find multiplication facts in the 9 times-table but not be able to use these to find the related division facts.

Key questions

- How could you use the 10 times-table to work out the 9 times-table?
- If you know a multiplication sentence, what division sentences can you find?
- How can you tell if a number is a multiple of 9?
- How can you use the 3 times-table to work out facts in the 9 times-table?

Possible sentence stems

- $\text{_____} \times 9 = \text{_____} \times 9 + \text{_____} \times 9$
- $\text{_____} \times 9 = \text{_____} - \text{_____} = \text{_____}$
- $\text{_____} \times 9 = \text{_____}$, so $\text{_____} \div 9 = \text{_____}$
- Multiplying by 9 is the same as multiplying by _____ and then multiplying by _____ again.

National Curriculum links

- Recall multiplication and division facts for multiplication tables up to 12×12
- Recognise and use factor pairs and commutativity in mental calculations

9 times-table and division facts

Key learning

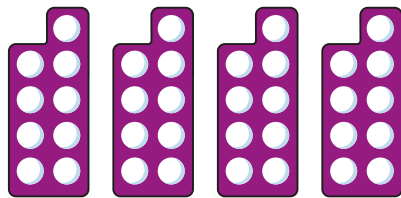
- Complete the sequence counting in 9s.

18, 27, _____, 45, 54, _____, 72, 81, _____, _____, 108

- Which of the numbers are multiples of 9?

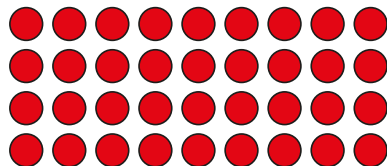
54	108	18	24	9
67	72	37	45	

- Write the multiplication fact to work out the total value of the number pieces.

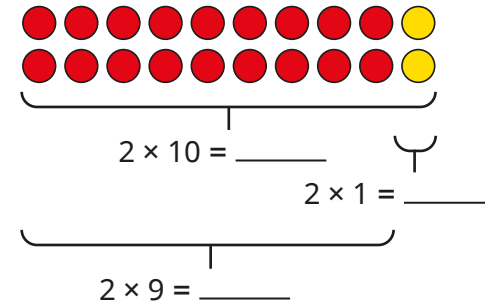


Write a division fact that you can see from the number pieces.

- Write the fact family to match the array.



- Use the array to complete the number sentences.



Use this method to complete the number sentences.

- ▶ $3 \times 10 = \underline{\quad}$ and $3 \times 1 = \underline{\quad}$ so $3 \times 9 = \underline{\quad}$
- ▶ $4 \times 10 = \underline{\quad}$ and $4 \times 1 = \underline{\quad}$ so $4 \times 9 = \underline{\quad}$
- ▶ $7 \times 10 = \underline{\quad}$ and $7 \times 1 = \underline{\quad}$ so $7 \times 9 = \underline{\quad}$

- Match the inverse operations.

$7 \times 9 = 63$

$108 \div 9 = 12$

$3 \times 9 = 27$

$81 \div 9 = 9$

$9 \times 9 = 81$

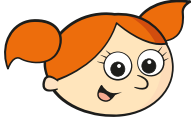


$27 \div 9 = 3$

$12 \times 9 = 108$


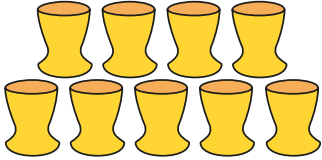

$63 \div 9 = 7$

9 times-table and division facts

Reasoning and problem solving

Alex has 63 flowers and some vases.
She puts 9 flowers into each vase.
How many vases does she need?


Teddy has 63 flowers. He has 9 vases.
He puts an equal number of flowers in each vase.
How many flowers does he put in each vase?
What is the same about these questions? What is different?

7 vases

7 flowers

Mo is thinking of two numbers.

The sum of my numbers is 17
The product of my numbers is 72



What are Mo's numbers?

8 and 9

Is this statement always true, sometimes true or never true?

Multiples of 9 are also multiples of 6

Explain your answer.

sometimes true

The 3, 6 and 9 times-tables

Notes and guidance

In this small step, children make links between the 3, 6 and 9 times-tables to deepen their understanding and embed fluency with these times-tables.

This is done by exploring the structure of the times-tables using resources such as arrays and hundred squares, as well as via tasks that require children to reason and explore number facts to look for structural patterns.

On completion of this step, children should be confident with their 2, 3, 4, 5, 6, 8, 9 and 10 times-tables before moving on to look at the remaining times-tables later in the block.

Things to look out for

- Children may see the pattern of doubling 3 times-table facts to find 6 times-table facts, then make the mistake of assuming that they can double the 6 times-table facts to find 9 times-table facts.
- Children may rely on reciting the times-tables, rather than using known facts at other points in the times-tables to help them.
- Even when children are secure in multiplication facts, they may not be confident with the corresponding divisions.

Key questions

- What links can you see between the 3 and 6 times-tables?
- What links can you see between the 3 and 9 times-tables?
- What other times-tables can you use to help find the multiplication facts?
- If you know one multiplication fact, what other multiplication fact do you know? What division facts do you know?
- How do you know if a number is a multiple of 3/6/9?

Possible sentence stems

- Double _____ $\times 3 =$ _____ $\times 6$
- Triple _____ $\times 3 =$ _____ $\times 9$
- 3 lots of _____ and 6 lots of _____ = 9 lots of _____
- _____ $\times 3 \times 3 =$ _____ \times _____

National Curriculum links

- Recall multiplication and division facts for multiplication tables up to 12×12
- Recognise and use factor pairs and commutativity in mental calculations

The 3, 6 and 9 times-tables

Key learning

- Here is a hundred square.
 - ▶ Circle the multiples of 3 in one colour.
 - ▶ Circle the multiples of 6 in another colour.
 - ▶ Circle the multiples of 9 in a third colour.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

What do you notice?

- Here are three number tracks for the 3, 6 and 9 times-tables. Complete the number tracks.

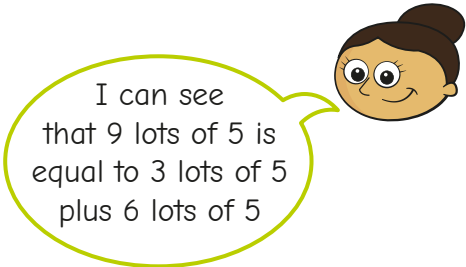
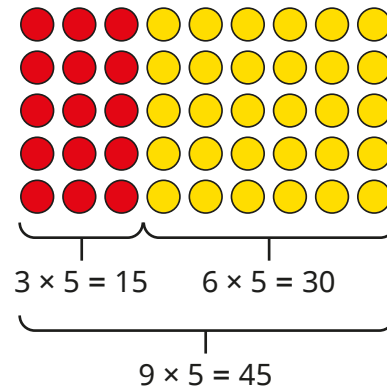
3	6	9	12							33	
---	---	---	----	--	--	--	--	--	--	----	--

6	12	18							60		
---	----	----	--	--	--	--	--	--	----	--	--

9				45							
---	--	--	--	----	--	--	--	--	--	--	--

What do you notice?

- Dora has made an array to show 9×5



Draw and label an array to show that $9 \times 4 = 3 \times 4 + 6 \times 4$

- What does the bar model show about the connection between the 3 times-table and the 9 times-table?



- Tommy has 9 bags of 6 apples. Write a multiplication to find the total number of apples. Write the fact family for this multiplication.

The 3, 6 and 9 times-tables

Reasoning and problem solving

Is the statement true or false?

All multiples of 3 are also multiples of 6 and 9

False

Explain your answer.



Ron is thinking of a 2-digit number.



My number is a multiple of 3, 6 and 9



There are more tens than ones.

If you halve this number, you get an even number.

What is Ron's number?

72



Scott buys 5 pairs of socks, 4 pairs of shorts and 6 T-shirts.

How much does Scott spend?

£93

Tom and Aisha have £36 between them.

Tom has twice as much money as Aisha.

Tom shares his money between 6 friends.

How much does each friend get?



£4

Multiply and divide by 7

Notes and guidance

In this small step, children use their knowledge of multiples and count in 7s to make the link between repeated addition and multiplication.

Children apply their knowledge of equal groups and use a range of concrete and pictorial representations to deepen their understanding of multiplying by 7. They also draw on ideas from previous steps to explore flexible partitioning to show, for example, $8 \times 7 = 5 \times 7 + 3 \times 7$ or $8 \times 7 = 8 \times 5 + 8 \times 2$

Children also explore dividing by 7 through sharing into 7 equal groups and grouping into 7s.

Things to look out for

- Children may need support to use the multiplication facts that they are confident in to find the ones they do not know as well.
- Children may not be able to identify which number in a number sentence corresponds with which number in a context.
- Children may find all multiplication facts by starting from 1×7 and then reciting their times-table facts, rather than using facts they know to find the facts they do not know.

Key questions

- How many equal groups are there?
- How many lots of 7 do you have?
- How many groups of 7 are there in _____?
- What can you partition _____ into to help you multiply _____ by 7?
- If you know this, what else do you know?
- How can you use the 5/6/8 times-table to find a fact in the 7 times-table?

Possible sentence stems

- _____ \times 7 = _____ \times 7 + _____ \times 7
- _____ \times 7 = _____ \times 8 - _____ = _____
- There are 7 groups of _____ in _____

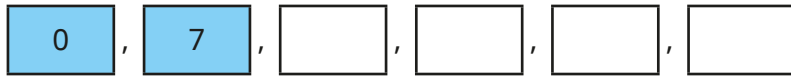
National Curriculum links

- Count in multiples of 6, 7, 9, 25 and 1,000
- Recall multiplication and division facts for multiplication tables up to 12×12

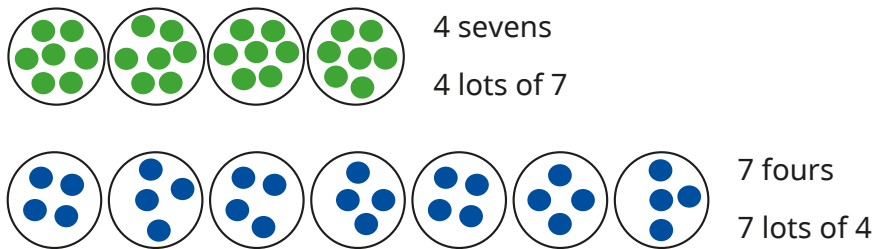
Multiply and divide by 7

Key learning

- Count in 7s to continue the sequence.

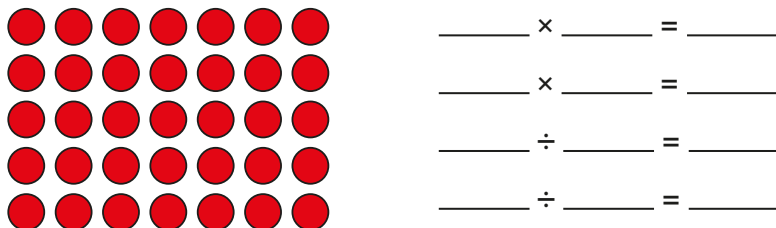


- Rosie draws a picture to represent 7×4 in two different ways.

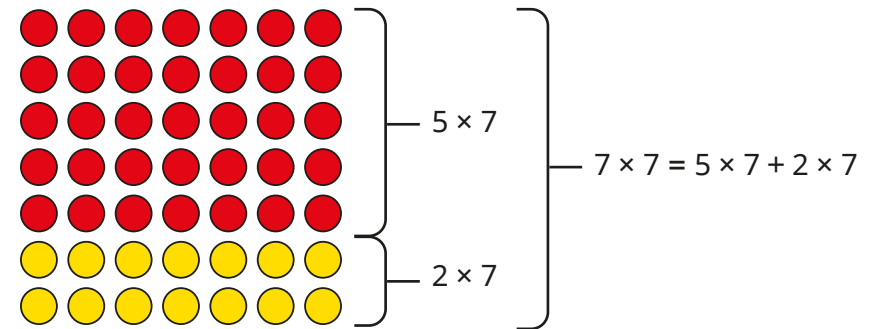


Use Rosie's method to represent 7×6 in two ways.

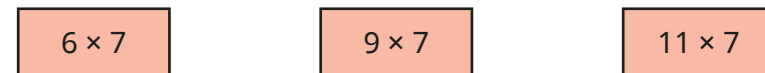
- Write two multiplications and two divisions shown by the array.



- Amir is using partitioning to help him work out 7×7



Use Amir's method to work out the multiplications.



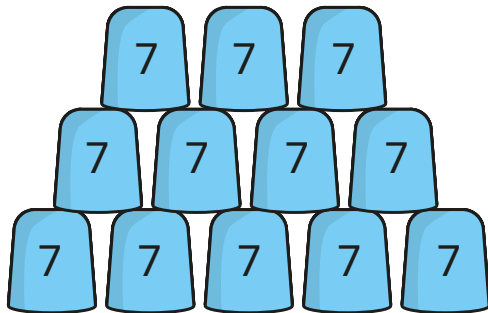
- 7 children can sit around one table.
How many children can sit around 5 tables?
- 7 children can sit around one table.
There are 63 children.
How many tables are needed?

Multiply and divide by 7

Reasoning and problem solving

Three children are playing a game.

They score 7 points for every cup they knock down.



Here are their scores.

Esther	56
Brett	77
Alex	28

How many cups did each child knock down?



Esther: 8 cups
Brett: 11 cups
Alex: 4 cups

Dexter is thinking of a number less than 70



My number is a multiple of 2, 4 and 7

28 or 56

What number could Dexter be thinking of?

Show that



$$9 \times 7 = 9 \times 8 - 9$$

any correct array

Draw an array to help you explain your answer.

7 times-table and division facts

Notes and guidance

In this small step, children bring together their knowledge of multiplying and dividing by 7 in order to become more fluent in the 7 times-table.

Children construct fact families and use concrete and pictorial representations to make links between multiplication and division. It is important that children understand the structure of the multiplication table and can derive unknown facts from known facts. Children explore links between multiplication tables, investigating how this can help with mental strategies for calculation, such as $9 \times 7 = 9 \times 5 + 9 \times 2$. This step could also be an opportunity to use the 6 and 8 times-tables to derive the 7 times-table, for example $9 \times 7 = 9 \times 8 - 9$ or $9 \times 7 = 9 \times 6 + 9$. Drawing arrays is a useful way of helping children to see these links.

Things to look out for

- Children may need support to use the multiplication facts that they are confident in to find the ones that they do not know as well.
- Children may find all multiplication facts by starting from 1×7 and then reciting their times-table facts, rather than using facts they know to find the facts they do not know.

Key questions

- How many lots of 7 do you have?
- What is the same and what is different about the number facts?
- How does the 7 times-table help you work out the answers?
- What strategies can you use to work out a 7 times-table fact that you do not yet know? What other times-tables can you use?

Possible sentence stems

- _____ \times 7 = _____ \times 5 + _____ \times 2
- _____ \times 7 = _____ \times 8 - _____
- _____ \times 7 = _____ \times 6 + _____
- There are 7 groups of _____ in _____
- There are _____ groups of 7 in _____

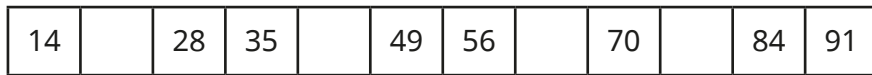
National Curriculum links

- Count in multiples of 6, 7, 9, 25 and 1,000
- Recall multiplication and division facts for multiplication tables up to 12×12

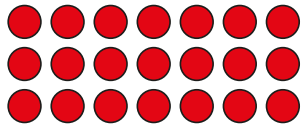
7 times-table and division facts

Key learning

- Complete the number track.



- Complete the fact family to match the array.



$\underline{\quad} \times \underline{\quad} = \underline{\quad}$
 $\underline{\quad} \times \underline{\quad} = \underline{\quad}$
 $\underline{\quad} \div \underline{\quad} = \underline{\quad}$
 $\underline{\quad} \div \underline{\quad} = \underline{\quad}$

- Match the inverse operations.

$8 \times 7 = 56$

$28 \div 7 = 4$

$6 \times 7 = 42$

$84 \div 7 = 12$

$12 \times 7 = 84$

$42 \div 7 = 6$

$4 \times 7 = 28$

$56 \div 7 = 8$

- Complete the multiplications.

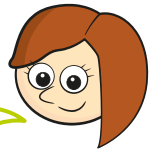
$\triangleright 11 \times 7 = \underline{\quad}$ $\triangleright 7 \times 9 = \underline{\quad}$ $\triangleright 70 = \underline{\quad} \times 7$
 $\triangleright \underline{\quad} \times 7 = 21$ $\triangleright 7 \times \underline{\quad} = 35$ $\triangleright \underline{\quad} = 1 \times 7$

- Dexter, Rosie and Whitney are working out 3×7 and explaining their methods.



Dexter

I did
 $3 \times 5 + 3 \times 2$
 $15 + 6 = 21$



Rosie

I counted in
 3s seven times:
 3, 6, 9, 12, 15, 18, 21



Whitney

I know 3×6 is 18,
 so I need to add 1 more lot
 of 3, which gives 21

Whose method do you prefer?

Is one method more efficient than the others?

Choose the method that you prefer to work out the multiplications.

9×7

5×7

12×7

7 times-table and division facts

Reasoning and problem solving

Is the statement true or false?

$$6 \times 7 = 5 \times 7 + 5$$

False

Explain your reasoning.



Whitney is thinking of a number.



My number is a multiple of 7

It is one more than a multiple of 5

It is less than 100



21, 56 or 91

What could Whitney's number be?

7 friends are going to the theme park and having pizza.



Tickets to the theme park cost £30 each.

A pizza costs £11

How much does it cost in total for all 7 friends to go to the theme park and to each have one pizza?

£287

Children are arranged into rows of 7



There are 5 girls and 2 boys in each row.



60

There are 84 children in total.

How many girls are there?

11 times-table and division facts

Notes and guidance

In this small step, children build on their knowledge of the 1 and 10 times-tables to explore the 11 times-table. They recognise that they can partition 11 into 10 and 1 and use known facts to support their understanding, for example $7 \times 11 = 7 \times 10 + 7 \times 1 = 77$

They use a range of concrete and pictorial representations to deepen their understanding of multiplying by 11 and to make links between multiplying and dividing by 11. They explore dividing by 11 through sharing into 11 equal groups and grouping into 11s.

At this stage, children should already know the majority of facts from other times-tables, so highlighting the importance of commutativity is key in this step.

Things to look out for

- Children may need support to use the multiplication facts that they are confident in to find the ones that they do not know as well.
- Children may not realise that 110, 121, 132 and so on are multiples of 11, as the previous multiples of 11 all have repeated digits, for example 66, 77, 88

Key questions

- How many equal groups are there?
- How many lots of 11 do you have?
- How many groups of 11 are there in _____?
- What can you partition 11 into to help you?
- How can you use base 10 to work out _____ \times 11?
- How can you use place value counters to work out _____ \div 11?
- How can you show this using an array?

Possible sentence stems

- _____ \times 11 = _____
- _____ \times 11 = _____ \times 10 + _____ \times 1
- There are 11 groups of _____ in _____
- There are _____ groups of 11 in _____

National Curriculum links

- Recall multiplication and division facts for multiplication tables up to 12×12
- Recognise and use factor pairs and commutativity in mental calculations

11 times-table and division facts

Key learning

- Complete the sentences.

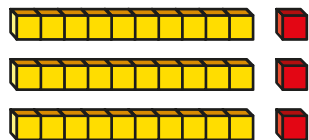


$2 \times 10 = \underline{\quad}$ $2 \times 1 = \underline{\quad}$

2 lots of 10 doughnuts = $\underline{\quad}$ 2 lots of 1 doughnut = $\underline{\quad}$

$2 \times 10 + 2 \times 1 = 2 \times 11 = \underline{\quad}$ There are $\underline{\quad}$ doughnuts.

- Tommy is using base 10 to help him work out 3×11



$3 \times 11 = 33$

Use Tommy's method to work out the multiplications.

5×11

8×11

7×11

10×11

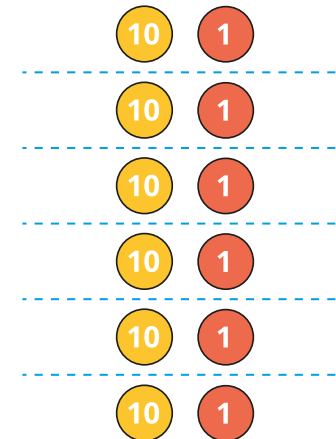
6×11

12×11

What do you notice?

- There are 11 players in a football team.
How many players are there in 4 teams?

- Nijah is using place value counters to help her work out $66 \div 11$



Use Nijah's method to work out the divisions.

$99 \div 11$

$55 \div 11$

$22 \div 11$

- 11 children can sit around one table.
There are 121 children.
How many tables are needed?

11 times-table and division facts

Reasoning and problem solving

Here is one batch of muffins.



132

- strawberry: 33
- vanilla: 33
- chocolate: 44
- toffee: 22

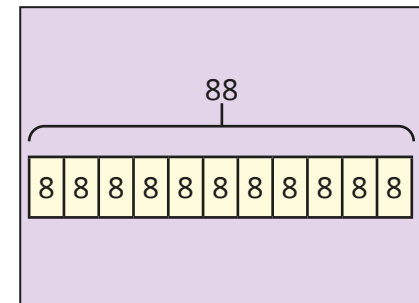
Amir bakes 11 batches of muffins.
How many muffins does he bake altogether?

In each batch, there are
3 strawberry, 3 vanilla, 4 chocolate
and 2 toffee muffins.

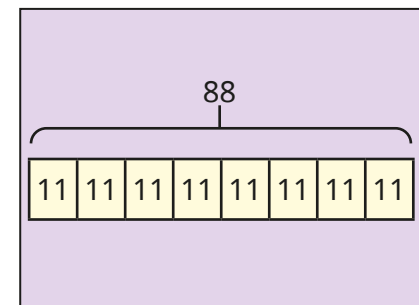
How many of each type of muffin does
Amir have in 11 batches?

Match the word problems to the bar models.

Dora has 88 footballs.
She shares them equally
into 11 bags.
How many footballs
are in each bag?



Dora has 88 footballs.
She wants to put them
into bags with 11 footballs
in each bag.
How many bags does
she use?



Explain your reasoning.



The first problem goes with the first bar model, and the second problem with the second bar model.

12 times-table and division facts

Notes and guidance

In this small step, children build on their knowledge of the 2 and 10 times-tables to explore the 12 times-table. They recognise that they can partition 12 into 10 and 2 and use known facts to support their understanding, for example $7 \times 12 = 7 \times 10 + 7 \times 2 = 84$. They also build on their knowledge of the 6 times-table, recognising that multiplying by 12 is the same as multiplying by 6 and then doubling.

Children use a range of concrete and pictorial representations to deepen their understanding of multiplying by 12 and to make links between multiplying and dividing by 12. They explore dividing by 12 through sharing into 12 equal groups and grouping into 12s.

At this stage, children should already know multiplication facts from other times-tables, so highlighting the importance of commutativity is key in this step.

Things to look out for

- Children may need support to use known multiplication facts to find new ones.
- Children may find all multiplication facts by starting from 1×12 and then reciting their times-table facts, rather than using facts that they know.

Key questions

- How many equal groups are there?
- How many lots of 12 do you have?
- How many groups of 12 are there in _____?
- What can you partition 12 into to help you?
- How can you use base 10 to work out _____ \times 12?
- How can you use place value counters to work out _____ \div 12?

Possible sentence stems

- _____ \times 12 = _____ \times 10 + _____ \times 2
- _____ \times 12 = double _____ \times 6
- There are 12 groups of _____ in _____
- There are _____ groups of 12 in _____

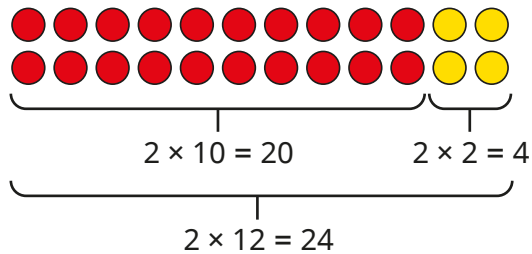
National Curriculum links

- Recall multiplication and division facts for multiplication tables up to 12×12
- Recognise and use factor pairs and commutativity in mental calculations

12 times-table and division facts

Key learning

- Jack has made an array to help him work out 2×12 . He has partitioned 12 into 10 and 2.

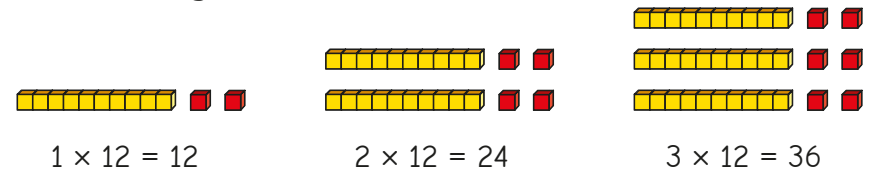


Use Jack's method to work out the multiplications.

5×12	8×12	7×12
10×12	6×12	12×12

- There are 12 people on a lacrosse team. There are 6 teams in a tournament. How many players are there altogether?
- A box holds 12 eggs. How many boxes are needed for 36 eggs?

- Sam is building the 12 times-table.



Use base 10 to help you complete the multiplications.

- $12 \times 5 = \underline{\quad}$ $5 \times 12 = \underline{\quad}$ $48 \div 12 = \underline{\quad}$ $84 \div 12 = \underline{\quad}$
 $12 \times \underline{\quad} = 120$ $12 \times \underline{\quad} = 132$ $\underline{\quad} \div 12 = 8$ $\underline{\quad} = 9 \times 12$

- Write $<$, $>$ or $=$ to make each statement correct.

4×12 ○ 6×12	$7 \times 10 + 7 \times 2$ ○ 7×12
8×12 ○ 12×8	$132 \div 12$ ○ 12×11
$48 \div 12$ ○ $72 \div 12$	9×12 ○ $9 \times 6 \times 2$

12 times-table and division facts

Reasoning and problem solving

Is this statement always true, sometimes true or never true?

When you multiply any whole number by 12, the answer will always be even.

always true

Explain your answer.



Tiny is thinking of a number less than 100

- It is a multiple of 7
- It is one less than a multiple of 12

35

What is Tiny's number?

Complete the table.

×	3	6	12
3			
6			
12			

9, 18, 36
18, 36, 72
36, 72, 144

What connections do you notice between the 3, 6 and 12 times-tables?



Here are the prices of tickets to see a play.

Adult	Child
£12	£6

What possible combination of adults and children could attend if they spend £60?

How many possibilities are there?

6

Adult	Children
5	0
4	2
3	4
2	6
1	8
0	10

Multiply by 1 and 0

Notes and guidance

In this small step, children explore the effect of multiplying by 1. They notice that when they multiply a number by 1, the result will always be the number itself.

This small step also focuses on multiplying by zero. Children learn that when multiplying any number by zero the result is always zero.

A common misconception with this small step is that children confuse the result of multiplying by zero with multiplying by 1. Ensure pictorial representations are used to address this misconception, so that children can see that 4×0 is the same as 4 lots of zero, which is equal to zero.

Things to look out for

- Children may use addition instead of multiplication, for example $1 \times 1 = 2$ and $8 \times 1 = 9$
- Children may confuse the result of multiplying by zero with multiplying by 1
- When working out a longer multiplication, for example $3 \times 4 \times 5 \times 0$, children may start working from left to right rather than realising that as they are multiplying by zero the answer must be zero.

Key questions

- What does “zero” mean? How can you multiply by zero?
- What do you notice about the results of multiplying numbers by zero?
- What does “multiplying by 1” mean?
- What do you notice about the results of multiplying numbers by 1?
- What is the same and what is different about multiplying by 1 and multiplying by zero?

Possible sentence stems

- Any number multiplied by zero is equal to _____
- Any number multiplied by 1 is equal to _____
- _____ groups of one = _____
- _____ groups of zero = _____

National Curriculum links

- Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers

Multiply by 1 and 0

Key learning

- Write a multiplication to work out the total number of pears.

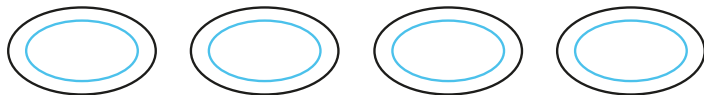


_____ × _____ = _____



_____ × _____ = _____

- There are 4 plates.
Each plate has zero apples on it.



How many apples are there in total?

Complete the multiplication.

$4 \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

- Complete the multiplications.

$7 \times 1 = \underline{\hspace{1cm}}$
 $7 \times 0 = \underline{\hspace{1cm}}$
 $1 \times \underline{\hspace{1cm}} = 12$
 $12 \times \underline{\hspace{1cm}} = 0$
 $1 \times 7 = \underline{\hspace{1cm}}$
 $0 \times 7 = \underline{\hspace{1cm}}$
 $\underline{\hspace{1cm}} \times 1 = 12$
 $0 = \underline{\hspace{1cm}} \times 12$

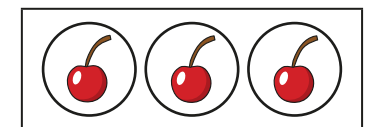
- Which calculations have an answer of zero?

48×1 0×38 1×1 0×0 4×0 10×1

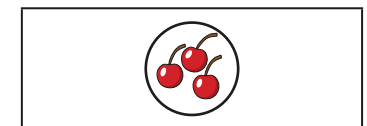
What do you notice?

- Match the statements to the pictures.

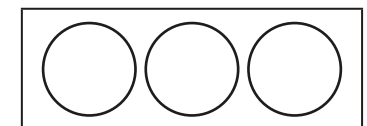
3 lots of 0



3 lots of 1

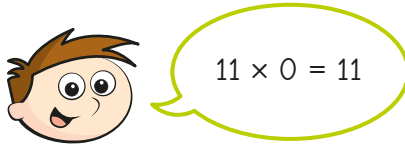


1 lot of 3




Multiply by 1 and 0

Reasoning and problem solving

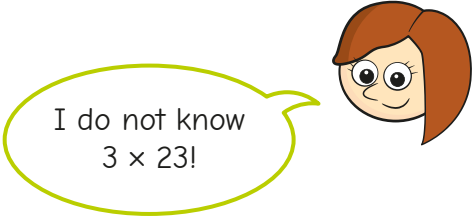


Do you agree with Teddy?
Explain your answer.

No



Rosie is working out $3 \times 23 \times 0 \times 9$



Explain why Rosie does not need to multiply the numbers one by one.

There is a zero in the calculation.
Any number multiplied by zero is zero.

$3 + 0 = \underline{\quad}$

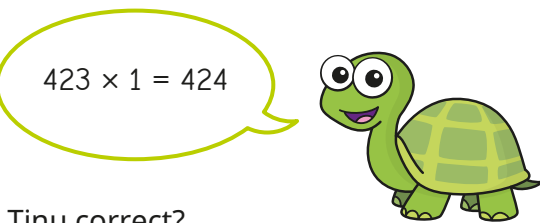
$3 - 0 = \underline{\quad}$

$3 \times 0 = \underline{\quad}$

Which is the odd one out?
Explain your choice.


3×0
It is the only one with zero as the answer.

Tiny is multiplying numbers.



Is Tiny correct?
Explain your answer.

No



Divide a number by 1 and itself

Notes and guidance

In this small step, children apply their knowledge of division and explore what happens to a number when they divide it by 1 or itself.

Children can sometimes confuse the result of dividing a number by 1 with dividing a number by itself. Ensure concrete and pictorial representations are used to address this misconception, including examples that involve both structures of division. Stem sentences can be used to encourage children to see this, for example: 5 grouped into 5s is equal to 1 ($5 \div 5 = 1$) and 5 grouped into 1s is equal to 5 ($5 \div 1 = 5$).

Following on from the previous small step, children may try to divide a number by zero and it should be highlighted that this is not possible.

Things to look out for

- Children may assume that division is commutative and think that $12 \div 1 = 1 \div 12$
- Children may confuse the result of dividing a number by 1 with dividing the number by itself.
- Children may think a number divided by itself is zero.

Key questions

- How many equal groups of _____ can you make?
- What is _____ shared equally into 1 group?
- What is _____ grouped into groups of 1?
- What is the same and what is different about multiplying by 1 and dividing by 1?
- What is the same and what is different about dividing a number by 1 and dividing a number by itself?

Possible sentence stems

- When you divide a number by itself, the answer is ...
- When you divide a number by _____, the number remains the same.
- There are _____ 1s in _____
- There is 1 _____ in _____

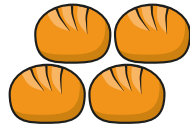
National Curriculum links

- Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers

Divide a number by 1 and itself

Key learning

- Complete the sentences.



4 shared into 1 equal group is equal to _____

4 grouped into groups of 1 is equal to _____

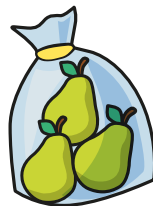
$4 \div 1 = \underline{\quad}$

- Here is a bag of 3 pears.

The pears are shared between 3 children.

How many pears does each child get?

$3 \div 3 = \underline{\quad}$



- Write a division sentence for each statement.

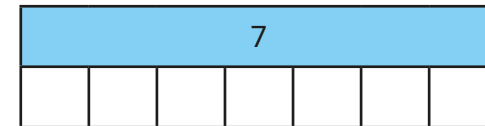
Use counters to help you.

- ▶ 4 counters shared into 4 groups
- ▶ 9 counters grouped into ones
- ▶ 7 counters shared into 1 group
- ▶ 6 counters grouped into sixes

- Dani bakes 7 cookies.

She shares them equally between her 7 friends.

How many cookies does each friend get?



$7 \div \underline{\quad} = \underline{\quad}$

- A bag can hold 5 apples.

Ron has 5 apples.

How many bags can he fill?

- $8 \div 8 = 1$

$12 \div 12 = 1$

What do you notice?

What other divisions can you write with an answer of 1?

- Which of the divisions have an answer of 1?

$100 \div 100$


$2 \div 1$

$10 \div 5$

$2 \div 2$

Divide a number by 1 and itself


Reasoning and problem solving



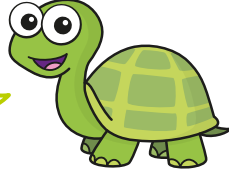
8×1 is greater than $8 \div 1$ because I am multiplying.

No

Do you agree with Ron?
Explain your reasoning.




$12 \div 1 \div 3$



I only need to divide 12 by 3 to work out the answer.

Yes

Do you agree with Tiny?
Explain your answer.



Without working out the divisions, write $<$, $>$ or $=$ to compare the statements.


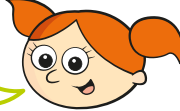
$8 \div 1$ ○ $7 \div 1$

$6 \div 6$ ○ $5 \div 5$

$4 \div 4$ ○ $4 \div 1$

$>$
 $=$
 $<$


Explain your reasoning.

25 divided by 1 is the same as 1 divided by 25.

No

Do you agree with Alex?
Explain your answer.



Multiply three numbers

Notes and guidance

In this small step, children apply their knowledge of multiplication to multiply three numbers together.

They are introduced to the idea of the associative law (but do not need to know it by name), which focuses on the fact that it does not matter how they group the numbers when they multiply. For example, $4 \times 5 \times 2 = (4 \times 5) \times 2 = 20 \times 2 = 40$ or $4 \times (5 \times 2) = 4 \times 10 = 40$

Encourage children to link this idea to commutativity and change the order of the numbers to group them more efficiently.

Counters and cubes are effective concrete resources to use during this step to support children's understanding of the associative law.

Things to look out for

- Children may need support ordering the numbers to group them more efficiently.
- If children are not confident with their times-table facts, they may struggle with multiplying three numbers.
- Children may automatically work from left to right without looking at the most efficient way to complete a calculation.

Key questions

- Do you have to multiply the numbers from left to right?
- Which pair(s) of numbers do you know the product of?
- How will you decide which order to do the multiplication in?
- What is the same about these calculations/arrays?
- Which order do you find easier to calculate efficiently?
- If you worked out the calculation in a different order, would you get a different answer? Why/why not?

Possible sentence stems

- I am going to work out $____ \times ____ \text{ first, because ...}$
- To work out $____ \times ____ \times ____ \text{, I can first calculate } ____ \times ____ \text{ and then multiply the answer by } ____$
- If $____ \times ____ = ____ \text{, then } ____ \times ____ \times ____ = ____$

National Curriculum links

- Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers

Multiply three numbers

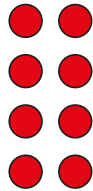
Key learning

- Complete the workings.

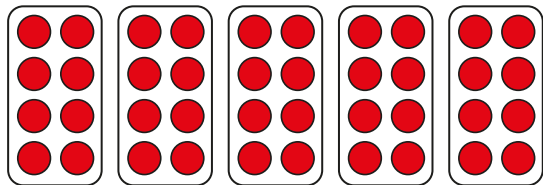
$2 \times 4 = \underline{\quad}$
 $2 \times 4 = \underline{\quad}$
 $2 \times 4 = \underline{\quad}$

$3 \times 2 \times 4 = 3 \times 8 = \underline{\quad}$

- How does the array show 4×2 ?



How does the array show $(4 \times 2) \times 5$?



Make an array to show $(5 \times 2) \times 4$

What do you notice?

- Find the products.

$5 \times 2 \times 6$

$8 \times 4 \times 5$

$2 \times 8 \times 6$

- Alex and Teddy are working out $6 \times 5 \times 2$

Alex	Teddy
$6 \times 5 \times 2 = 6 \times 5 \times 2$ $= 30 \times 2$ $= 60$	$6 \times 5 \times 2 = 6 \times 5 \times 2$ $= 6 \times 10$ $= 60$

Whose method do you prefer?

Is one method more efficient than the other?

Choose the method you prefer to work out the calculations.

$7 \times 4 \times 2$

$3 \times 5 \times 4$

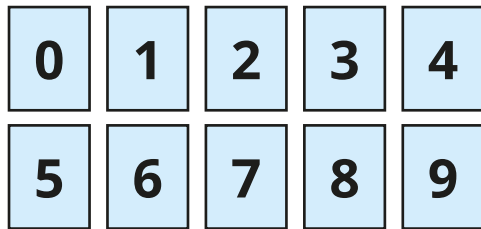
$3 \times 4 \times 8$

- In a field there are 7 animal pens.
In each pen there are 4 rabbit hutches.
In each rabbit hutch there are 3 rabbits.
How many rabbits are there in total?

Multiply three numbers

Reasoning and problem solving

Choose three digit cards.



Find the product of your digit cards.

How many different calculations can you make?

What is the most efficient order to use to work out the product?



Answers will vary depending on the numbers chosen.

Kim rolls a 10-sided dice three times.

The product of her numbers is 40

What numbers could she have rolled?

Compare answers with a partner.



multiple possible answers, e.g.

1, 4, 10

1, 5, 8

2, 4, 5

2, 2, 10



Is the statement true or false?

$$9 \times 8 = 9 \times 4 \times 2$$

True

Explain your reasoning.



Which calculation is the odd one out?

$$4 \times 10 \times 2$$

$$4 \times 4 \times 5$$

$$5 \times 4 \times 2$$

$$5 \times 2 \times 8$$

Children can choose any card with the correct justification.

Explain your reasoning.

